Supplemental file for “A Weighted Biobjective Transformation Technique for Locating Multiple Optimal Solutions of Nonlinear Equation Systems”

Wenyin Gong, Yong Wang, *Member, IEEE*, Zhihua Cai, and Shengxiang Yang, *Senior Member, IEEE*

S-I. Thirty-eight Test Instances

1) F01:

$$\begin{cases} x_1^2 + x_2^2 - 1 = 0 \\ x_1 - x_2 = 0 \end{cases} \tag{1}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 2$. It has two optimal solutions: (-0.707107, -0.707107) and (0.707107, 0.707107) [1].

2) F02:

$$\begin{cases} \sum_{i=1}^{n} x_i^2 - 1 = 0 \\ |x_1 - x_2| + \sum_{i=3}^{n} x_i^2 = 0 \end{cases} \tag{2}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 20$. It has two optimal solutions: (-0.707107, -0.707107, 0, $\cdots$, 0) and (0.707107, 0.707107, 0, $\cdots$, 0) [1].

3) F03:

$$\begin{cases} x_1 - \sin(5\pi x_2) = 0 \\ x_1 - x_2 = 0 \end{cases} \tag{3}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 2$. It has 11 optimal solutions as shown in Table S-I [1].

TABLE S-I
THE OPTIMAL SOLUTIONS OF F03

x_1	x_2
-0.924840	-0.924840
-0.866760	-0.866760
-0.562010	-0.562010
-0.428168	-0.428168
-0.187960	-0.187960
0.000000	0.000000
0.187960	0.187960
0.428168	0.428168
0.562010	0.562010
0.866760	0.866760
0.924840	0.924840

4) F04:

$$\begin{cases} x_1 - \cos(4\pi x_2) = 0 \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} \tag{4}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 2$. It has 15 optimal solutions as shown in Table S-II [1].

W. Gong and Z. Cai are with the School of Computer Science, China University of Geosciences, Wuhan 430074, China. (Email: wygong@cug.edu.cn; zhcai@cug.edu.cn)

Y. Wang is with the School of Information Science and Engineering, Central South University, Changsha 410083, China, and also with the Centre for Computational Intelligence (CCI), School of Computer Science and Informatics, De Montfort University, Leicester LE1 9BH, UK. (Email: ywang@csu.edu.cn)

S. Yang is with the Centre for Computational Intelligence (CCI), School of Computer Science and Informatics, De Montfort University, Leicester LE1 9BH, UK. (Email: syang@dmu.ac.uk)

TABLE S-II
THE OPTIMAL SOLUTIONS OF F04

x_1	x_2
0.416408	-0.909178
-0.561364	-0.827569
-0.724322	-0.689462
0.837812	-0.545959
0.886984	0.461799
-0.962322	-0.271914
-0.972855	-0.231415
1.000000	0.000000
-0.972855	0.231416
-0.962322	0.271914
0.886984	0.461799
0.837812	0.545959
-0.724322	0.689462
-0.561364	0.827569
0.416408	0.909178

5) F05:

$$\begin{cases} \cos(2x_1) - \cos(2x_2) - 0.4 = 0 \\ 2(x_2 - x_1) + \sin(2x_2) - \sin(2x_1) - 1.2 = 0 \end{cases} \tag{5}$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, n$, and $n = 2$. It has 13 optimal solutions as shown in Table S-III [2].

TABLE S-III
THE OPTIMAL SOLUTIONS OF F05

x_1	x_2
-9.268258	-8.931402
-8.744542	-7.164787
-6.126665	-5.789809
-5.602950	-4.023195
-2.985073	-2.648216
-2.461357	-0.881602
0.156520	0.493376
0.680236	2.259991
3.298113	3.634969
3.821828	5.401583
6.439705	6.776562
6.963421	8.543176
9.581298	9.918154

6) F06:

$$\begin{cases} x_1 - 0.25428722 - 0.18324757 x_4 x_3 x_9 = 0 \\ x_2 - 0.37842197 - 0.16275449 x_1 x_{10} x_6 = 0 \\ x_3 - 0.27162577 - 0.16955071 x_1 x_2 x_{10} = 0 \\ x_4 - 0.19807914 - 0.15585316 x_7 x_1 x_6 = 0 \\ x_5 - 0.44166728 - 0.19950920 x_7 x_6 x_3 = 0 \\ x_6 - 0.14654113 - 0.18922793 x_8 x_5 x_{10} = 0 \\ x_7 - 0.42937161 - 0.21180486 x_2 x_5 x_8 = 0 \\ x_8 - 0.07056438 - 0.17081208 x_1 x_7 x_6 = 0 \\ x_9 - 0.34504906 - 0.19612740 x_{10} x_6 x_8 = 0 \\ x_{10} - 0.42651102 - 0.21466544 x_4 x_8 x_1 = 0 \end{cases} \tag{6}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, n$, and $n = 10$. It has one optimal solution: (0.257833, 0.381097, 0.278745, 0.200669, 0.445251, 0.149184, 0.432010, 0.073403, 0.345967, 0.427326) [2].

7) F07:

$$\begin{cases} 100(x_1 - 0.25) = 0 \\ 100(x_1 \sin(4\pi x_2^2) + 0.75x_1 - 0.25) = 0 \end{cases} \tag{7}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 2$. It has eight optimal solutions as shown in Table S-IV.

TABLE S-IV
THE OPTIMAL SOLUTIONS OF F07

x_1	x_2
0.250000	-0.854337
0.250000	-0.721185
0.250000	-0.479471
0.250000	-0.141801
0.250000	0.141801
0.250000	0.479471
0.250000	0.721185
0.250000	0.854337

8) F08:

$$\begin{cases} 3.0 - x_1 x_3^2 = 0 \\ x_3 \sin(\pi/x_2) - x_3 - x_4 = 0 \\ -x_2 x_3 \exp(1.0 - x_1 x_3) + 0.2707 = 0 \\ 2x_1^2 x_3 - x_2^4 x_3 - x_2 = 0 \end{cases} \tag{8}$$

where $x_i \in [0, 5]$, $i = 1, \cdots, n$, and $n = 4$. It has one optimal solution: (3, 2, 1, 0) [3].

9) F09:

$$\begin{cases} (1-R)\left[\left(\frac{D}{10(1+\beta_1)} - x_1\right) \cdot \exp\left(\frac{10x_1}{1+\frac{10x_1}{\gamma}}\right)\right] - x_1 = 0 \\ (1-R)\left[\left(\frac{D}{10} - \beta_1 x_1 - (1+\beta_2)x_2\right) \cdot \exp\left(\frac{10x_2}{1+\frac{10x_2}{\gamma}}\right)\right] + x_1 - (1+\beta_2)x_2 = 0 \end{cases} \tag{9}$$

where $x_i \in [0, 1]$, $i = 1, \cdots, n$, $n = 2$, $R = 0.96$, $D = 22$, $\gamma = 1000$, and $\beta_1 = \beta_2 = 2$. It has seven optimal solutions as shown in Table S-V [3], [4], [5], [6].

TABLE S-V
THE OPTIMAL SOLUTIONS OF F09

x_1	x_2
0.042100	0.061813
0.042100	0.268723
0.266600	0.178430
0.266600	0.327267
0.266600	0.461111
0.042318	0.686779
0.719074	0.244197

10) F10:

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + 2x_2 + x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + x_2 + 2x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + x_2 + x_3 + 2x_4 + x_5 - 6.0 = 0 \\ x_1 x_2 x_3 x_4 x_5 - 1.0 = 0 \end{cases} \tag{10}$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, n$, and $n = 5$. It has three optimal solutions: (1, 1, 1, 1, 1), (0.916355, 0.916355, 0.916355, 0.916355, 1.418227), and (-0.579043, -0.579043, -0.579043, -0.579043, 8.895215) [7], [8].

11) F11:

$$\begin{cases} x_1 + x_2^4 x_4 x_6/4 + 0.75 = 0 \\ x_2 + 0.405\exp(1 + x_1 x_2) - 1.405 = 0 \\ x_3 - x_4 x_6/2 + 1.5 = 0 \\ x_4 - 0.605\exp(1 - x_3^2) - 0.395 = 0 \\ x_5 - x_2 x_6/2 + 1.5 = 0 \\ x_6 - x_1 x_5 = 0 \end{cases} \tag{11}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 6$. It has one optimal solution: (-1, 1, -1, 1, -1, 1) [8], [9].

12) F12:

$$\begin{cases} \sin(x_1^3) - 3x_1 x_2^2 - 1 = 0 \\ \cos(3x_1^2 x_2) - |x_2^3| + 1 = 0 \end{cases} \tag{12}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, n$, $n = 2$. It has 10 optimal solutions as shown in Table S-VI. This function is modified from [10].

TABLE S-VI
THE OPTIMAL SOLUTIONS OF F12

x_1	x_2
-1.810885	-0.349092
-1.810885	0.349092
-1.502221	-0.409077
-1.502221	0.409077
-1.791302	0.301926
-1.791302	-0.301926
-0.947268	0.785020
-0.947268	-0.785020
-0.213057	1.256845
-0.213057	-1.256845

13) F13:

$$\begin{cases} 4x_1^3 + 4x_1 x_2 + 2x_2^2 - 42x_1 - 14 = 0 \\ 4x_2^3 + 2x_1^2 + 4x_1 x_2 - 26x_2 - 22 = 0 \end{cases} \tag{13}$$

where $x_i \in [-5, 5]$, $i = 1, \cdots, n$, and $n = 2$. It has nine optimal solutions as shown in Table S-VII [11], [12].

TABLE S-VII
THE OPTIMAL SOLUTIONS OF F13

x_1	x_2
-0.127961	-1.953715
-0.270845	-0.923039
0.086678	2.884255
3.385154	0.073852
3.584428	-1.848127
3.000000	2.000000
-3.779310	-3.283186
-3.073026	-0.081353
-2.805118	3.131313

14) F14:

$$\begin{cases} -\sin(x1)\cos(x2) - 2\cos(x1)\sin(x2) = 0 \\ -\cos(x1)\sin(x2) - 2\sin(x1)\cos(x2) = 0 \end{cases} \tag{14}$$

where $x_i \in [0, 2\pi]$, $i = 1, \cdots, n$, and $n = 2$. It has 13 optimal solutions as shown in Table S-VIII [5], [11].

TABLE S-VIII
THE OPTIMAL SOLUTIONS OF F14

x_1	x_2
0.000000	0.000000
3.141593	0.000000
1.570796	1.570796
6.283185	0.000000
0.000000	3.141593
4.712389	1.570796
3.141593	3.141593
1.570796	4.712389
6.283185	3.141593
0.000000	6.283185
4.712389	4.712389
3.141593	6.283185
6.283185	6.283185

15) F15:

$$\left\{\begin{array}{rcl} x_1^2+x_2^2-1.0 & = & 0 \\ x_3^2+x_4^2-1.0 & = & 0 \\ x_5^2+x_6^2-1.0 & = & 0 \\ x_7^2+x_8^2-1.0 & = & 0 \\ 4.731\cdot 10^{-3}x_1x_3-0.3578x_2x_3-0.1238x_1+x_7 & & \\ -1.637\cdot 10^{-3}x_2-0.9338x_4-0.3571 & = & 0 \\ 0.2238x_1x_3+0.7623x_2x_3+0.2638x_1-x_7 & & \\ -0.07745x_2-0.6734x_4-0.6022 & = & 0 \\ x_6x_8+0.3578x_1+4.731\cdot 10^{-3}x_2 & = & 0 \\ -0.7623x_1+0.2238x_2+0.3461 & = & 0 \end{array}\right. \tag{15}$$

where $x_i \in [-1,1]$, $i=1,\cdots,n$, and $n=8$. It has 16 optimal solutions as shown in Table S-IX [7], [11], [12].

TABLE S-IX
THE OPTIMAL SOLUTIONS OF F15

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
0.1644	-0.9864	-0.9471	-0.3210	-0.9982	-0.0594	0.4110	0.9116
0.1644	-0.9864	-0.9471	-0.3210	-0.9982	0.0594	0.4110	-0.9116
0.1644	-0.9864	-0.9471	-0.3210	0.9982	-0.0594	0.4110	0.9116
0.1644	-0.9864	-0.9471	-0.3210	0.9982	0.0594	0.4110	-0.9116
0.1644	-0.9864	0.7185	-0.6956	-0.9980	-0.0638	-0.5278	0.8494
0.1644	-0.9864	0.7185	-0.6956	-0.9980	0.0638	-0.5278	-0.8494
0.1644	-0.9864	0.7185	-0.6956	0.9980	-0.0638	-0.5278	0.8494
0.1644	-0.9864	0.7185	-0.6956	0.9980	0.0638	-0.5278	-0.8494
0.6716	0.7410	-0.6516	-0.7586	-0.9625	-0.2711	-0.4376	0.8992
0.6716	0.7410	-0.6516	-0.7586	-0.9625	0.2711	-0.4376	-0.8992
0.6716	0.7410	-0.6516	-0.7586	0.9625	-0.2711	-0.4376	0.8992
0.6716	0.7410	-0.6516	-0.7586	0.9625	0.2711	-0.4376	-0.8992
0.6716	0.7410	0.9519	-0.3064	-0.9638	-0.2666	0.4046	0.9145
0.6716	0.7410	0.9519	-0.3064	-0.9638	0.2666	0.4046	-0.9145
0.6716	0.7410	0.9519	-0.3064	0.9638	0.2666	0.4046	-0.9145
0.6716	0.7410	0.9519	-0.3064	0.9638	-0.2666	0.4046	0.9145

16) F16:

$$\left\{\begin{array}{l} 4x_1^3-3x_1-\cos(x_2)=0 \\ \sin(x_1^2)-|x_2|=0 \end{array}\right. \tag{16}$$

where $x_i \in [-2,2]$, $i=1,\cdots,n$, and $n=2$. It has six optimal solutions as shown in Table S-X. This function is modified from [13].

TABLE S-X
THE OPTIMAL SOLUTIONS OF F16

x_1	x_2
-0.597167	-0.349098
-0.597167	0.349098
-0.442758	-0.194781
-0.442758	0.194781
0.964499	-0.801774
0.964499	0.801774

17) F17:

$$\left\{\begin{array}{l} x_i+\sum_{j=1}^{n}x_j-(n+1)=0 \quad i=1,\cdots,n-1 \\ \left[\prod_{j=1}^{n}x_j\right]-1=0 \end{array}\right. \tag{17}$$

where $x_i \in [-2,2]$, $i=1,\cdots,n$, and $n=20$. It has two optimal solutions: (1, $\cdots$, 1) and (0.994922, $\cdots$, 0.994922, 1.101551) [13].

18) F18:

$$x_i-\cos\left(2x_i-\sum_{j=1}^{n}x_j\right)=0 \quad i=1,\cdots,n \tag{18}$$

where $x_i \in [-1,1]$, $i=1,\cdots,n$, and $n=3$. It has seven optimal solutions as shown in Table S-XI [14].

TABLE S-XI
THE OPTIMAL SOLUTIONS OF F18

x_1	x_2	x_3
0.810561	0.810561	-0.625687
0.810561	-0.625687	0.810561
-0.625687	0.810561	0.810561
0.543850	0.995778	0.543850
0.543850	0.543850	0.995778
0.995778	0.543850	0.543850
0.739086	0.739086	0.739086

19) F19:

$$\left\{\begin{array}{l} x_1^2+x_2^2-2=0 \\ x_1^2+x_2^2/4-1=0 \end{array}\right. \tag{19}$$

where $x_i \in [-2,2]$, $i=1,\cdots,n$, and $n=2$. It has four optimal solutions as shown in Table S-XII. This function is modified from [15].

TABLE S-XII
THE OPTIMAL SOLUTIONS OF F19

x_1	x_2
-0.816497	-1.154701
0.816497	-1.154701
-0.816497	1.154701
0.816497	1.154701

20) F20:

$$\left\{\begin{array}{l} \exp\left(x_1^2+x_2^2\right)-3=0 \\ |x_2|+x_1-\sin\left(3(|x_2|+x_1)\right)=0 \end{array}\right. \tag{20}$$

where $x_i \in [-2,2]$, $i=1,\cdots,n$, and $n=2$. It has six optimal solutions as shown in Table S-XIII. This function is modified from [15].

TABLE S-XIII
THE OPTIMAL SOLUTIONS OF F20

x_1	x_2
-0.741152	-0.741152
-0.741152	0.741152
-0.256625	1.016246
-0.256625	-1.016246
-1.016246	-0.256625
-1.016246	0.256625

21) F21:

$$\left\{\begin{array}{l} -3.84x_1^2+3.84x_1-x_2=0 \\ -3.84x_2^2+3.84x_2-x_3=0 \\ -3.84x_3^2+3.84x_3-x_1=0 \end{array}\right. \tag{21}$$

where $x_i \in [0,1]$, $i=1,\cdots,n$, and $n=3$. It has eight optimal solutions as shown in Table S-XIV [16].

TABLE S-XIV
THE OPTIMAL SOLUTIONS OF F21

x_1	x_2	x_3
0.000000	0.000000	0.000000
0.488122	0.959435	0.149452
0.540304	0.953754	0.169399
0.959447	0.149373	0.487917
0.149440	0.488092	0.959440
0.953781	0.169343	0.540157
0.169254	0.539937	0.953788
0.739584	0.739584	0.739574

22) F22:

$$\begin{cases} x_1 + x_2 + x_3 - 1 = 0 \\ x_1 - x_2^3 = 0 \end{cases} \tag{22}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 3$. It has infinite optimal solutions [1].

23) F23:

$$\begin{cases} x_1^2 + x_3^2 - 1 = 0 \\ x_2^2 + x_4^2 - 1 = 0 \\ x_5 x_3^3 + x_6 x_4^3 = 0 \\ x_5 x_1^3 + x_6 x_2^3 = 0 \\ x_5 x_1 x_3^2 + x_6 x_4^2 x_2 = 0 \\ x_5 x_1^2 x_3 + x_6 x_2^2 x_4 = 0 \end{cases} \tag{23}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, and $n = 6$. It has infinite optimal solutions [1], [2].

24) F24:

$$\begin{cases} (x_k + \sum_{i=1}^{n-k-1} x_i x_{i+k}) x_n - c_k = 0 \quad 1 \le k \le n-1 \\ \sum_{i=1}^{n-1} x_i + 1 = 0 \end{cases} \tag{24}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, n$, $n = 20$, $c_k = 0$, and $k = 1, \cdots, n-1$. It has infinite optimal solutions [1], [2].

25) F25:

$$\begin{cases} x_2 + 2x_6 + x_9 + 2x_{10} - 10^{-5} = 0 \\ x_3 + x_8 - 3 \cdot 10^{-5} = 0 \\ x_1 + x_3 + 2x_5 + 2x_8 + x_9 + x_{10} - 5 \cdot 10^{-5} = 0 \\ x_4 + 2x_7 - 10^{-5} = 0 \\ 0.5140437 \cdot 10^{-7} x_5 - x_1^2 = 0 \\ 0.1006932 \cdot 10^{-6} x_6 - 2x_2^2 = 0 \\ 0.7816278 \cdot 10^{-15} x_7 - x_4^2 = 0 \\ 0.1496236 \cdot 10^{-6} x_8 - x_1 x_3 = 0 \\ 0.6194411 \cdot 10^{-7} x_9 - x_1 x_2 = 0 \\ 0.2089296 \cdot 10^{-14} x_{10} - x_1 x_2^2 = 0 \end{cases} \tag{25}$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, n$, and $n = 10$. It has infinite optimal solutions [2].

26) F26:

$$\begin{cases} 3x_1^2 + \sin(x_1 x_2) - x_3^2 + 2.0 = 0 \\ 2x_1^3 - x_2^2 - x_3 + 3.0 = 0 \\ \sin(2x_1) + \cos(x_2 x_3) + x_2 - 1.0 = 0 \end{cases} \tag{26}$$

where $x_1 \in [-5, 5]$, $x_2 \in [-1, 3]$, and $x_3 \in [-5, 5]$. It has two optimal solutions: (-0.064417, 2.090440, -1.370473) and (-0.032759, 1.264629, 1.400644) [17].

27) F27:

$$\begin{cases} 5x_1^9 - 6x_1^5 x_2^2 + x_1 x_2^4 + 2x_1 x_3 = 0 \\ -2x_1^6 x_2 + 2x_1^2 x_2^3 + 2x_2 x_3 = 0 \\ x_1^2 + x_2^2 - 0.265625 = 0 \end{cases} \tag{27}$$

where $x_1 \in [-0.6, 6]$, $x_2 \in [-0.6, 0.6]$, and $x_3 \in [-5, 5]$. It has 12 optimal solutions as shown in Table S-XV [13], [11].

TABLE S-XV
THE OPTIMAL SOLUTIONS OF F27

x_1	x_2	x_3
0.279855	0.432789	-0.014189
0.279855	-0.432789	-0.014189
-0.279855	0.432789	-0.014189
-0.279855	-0.432789	-0.014189
0.466980	0.218070	0.000000
-0.466980	0.218070	0.000000
0.466980	-0.218070	0.000000
-0.466980	-0.218070	0.000000
0.000000	0.515388	0.000000
0.000000	-0.515388	0.000000
0.515388	0.000000	-0.012446
-0.515388	0.000000	-0.012446

28) F28:

$$\begin{cases} x_1^2 - x_2 - 2 = 0 \\ x_1 + \sin\left(\frac{\pi}{2} x_2\right) = 0 \end{cases} \tag{28}$$

where $x_1 \in [0, 1]$ and $x_2 \in [-10, 0]$. It has two optimal solutions (0, -2) and (0.707660, -1.5) [3].

29) F29:

$$\begin{cases} x_1^2 + x_2^2 + x_1 + x_2 - 8 = 0 \\ x_1 |x_2| + x_1 + |x_2| - 5 = 0 \end{cases} \tag{29}$$

where $x_1 \in [0, 2.5]$ and $x_2 \in [-4, 6]$. It has four optimal solutions (0.404634, -3.271577), (2.403604, -0.762837), (1, 2), and (2, 1). This problem is modified from [18].

30) F30:

$$\begin{cases} x_1^2 - |x_2| + 1 + \frac{1}{9}|x_1 - 1| = 0 \\ x_2^2 + 5x_1^2 - 7 + \frac{1}{9}|x_2| = 0 \end{cases} \tag{30}$$

where $x_1 \in [-1, 1]$ and $x_2 \in [-10, 10]$. It has four optimal solutions (-0.814326, -1.864719), (0.861828, -1.758100), (-0.814326, 1.864719), and (0.861828, 1.758100). This problem is modified from [18].

31) F31:

$$\begin{cases} 0.5 \sin(x_1 x_2) - \frac{0.25}{\pi} x_2 - 0.5 x_1 = 0 \\ \left(1 - \frac{0.25}{\pi}\right) [\exp(2x_1) - e] + \frac{e}{\pi} x_2 - 2e x_1 = 0 \end{cases} \tag{31}$$

where $x_1 \in [0.25, 1]$ and $x_2 \in [1.5, 2\pi]$. It has two optimal solutions (0.299465, 2.836948) and (0.499966, 3.141589) [12], [19].

32) F32:

$$\begin{cases} x_1^{x_2} + x_2^{x_1} - 5x_1 x_2 x_3 - 85 = 0 \\ x_1^3 - x_2^{x_3} - x_3^{x_2} - 60 = 0 \\ x_1^{x_3} + x_3^{x_1} - x_2 - 2 = 0 \end{cases} \tag{32}$$

where $x_1 \in [3, 5]$, $x_2 \in [2, 4]$, and $x_3 \in [0.5, 2]$. It has one optimal solution (4, 3, 1) [20].

33) F33:

$$\begin{cases} x_1^3 - 3x_1x_2^2 - 1 = 0 \\ 3x_1^2x_2 - x_2^3 + 1 = 0 \end{cases} \tag{33}$$

where $x_1 \in [-1, -0.1]$ and $x_2 \in [-2, 2]$. It has two optimal solutions (-0.793701, -0.793701) and (-0.290515, 1.084215) [9].

34) F34:

$$\begin{cases} 0.1x_1 + \cos(2x_2) + 0.09240 = 0 \\ \sin(3x_3) + \sin(\frac{10x_1}{3}) + \log(2x_2) - 2.52x_3 + 0.08805 = 0 \\ 2(x_1 - 0.75)^2 + \sin(16\pi x_2 - \frac{\pi}{2}) - 3.26815 = 0 \end{cases} \tag{34}$$

where $x_1 \in [1, 2.5]$, $x_2 \in [0.2, 2]$, and $x_3 \in [0.1, 3]$. It has one optimal solution (1.852100, 0.926050, 0.617370) [21].

35) F35:

$$\begin{cases} 4x_1^3 - 3x_1 - x_2 = 0 \\ x_1^2 - x_2 = 0 \end{cases} \tag{35}$$

where $x_1 \in [-5, 1.5]$ and $x_2 \in [0, 5]$. It has three optimal solutions (-0.75, 0.5625), (0, 0), and (1, 1) [13].

36) F36:

$$\begin{cases} x_1^3 - 3x_1x_2^2 + a_1(2x_1^2 + x_1x_2) + b_1x_2^2 + c_1x_1 + a_2x_2 = 0 \\ 3x_1^2x_2 - x_2^3 - a_1(4x_1x_2 - x_2^2) + b_2x_1^2 + c_2 = 0 \end{cases} \tag{36}$$

where $a_1 = 25, b_1 = 1, c_1 = 2, a_2 = 3, b_2 = 4, c_2 = 5$, $x_1 \in [0, 2]$, and $x_2 \in [10, 30]$. It has two optimal solutions (1.6359718, 13.8476653) and (0.6277425, 22.2444123) [15].

37) F37:

$$\begin{cases} x_1^2 - x_1 - x_2^2 - x_2 + x_3^2 = 0 \\ \sin(x_2 - \exp(x_1)) = 0 \\ x_3 - \log|x_2| = 0 \end{cases} \tag{37}$$

where $x_1 \in [0, 2]$, $x_2 \in [-10, 10]$, and $x_3 \in [-1, 1]$. It has five optimal solutions shown in Table S-XVI. This problem is modified from [22].

TABLE S-XVI
THE OPTIMAL SOLUTIONS OF F37

x_1	x_2	x_3
0.825297	-0.859034	-0.151946
1.299490	0.525835	-0.642769
1.533662	-1.648068	0.499604
1.981360	-2.172180	0.775731
1.983283	0.983378	-0.016762

38) F38:

$$\begin{cases} x_1^4 + 4x_2^4 - 6.0 = 0 \\ x_1^2x_2 - 0.6787 = 0 \end{cases} \tag{38}$$

where $x_1 \in [-2, 2]$ and $x_2 \in [0, 1.1]$. It has four optimal solutions (-1.563533, 0.277628), (-0.789706, 1.088295), (1.563533, 0.277628), and (0.789706, 1.088295). This problem is modified from [23].

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S-II. Parameter Settings

TABLE S-R-I
Parameter settings for different methods.

Method	Parameter settings
A-WeB	$NP = 100, H = NP$
A-MONES	$NP = 100, H = NP$
A-MOMMOP	$NP = 100, H = NP$
A-MOBiDE	$NP = 100, H = NP$
NCDE	$NP = 100, F = 0.9, CR = 0.1$
NSDE	$NP = 100, F = 0.9, CR = 0.1$
LIPS	$NP = 100, w = 0.729843788$
R3PSO	$NP = 100, w = 0.729843788, c_1 = c_2 = 2.05$
Rep-SHADE	$NP = 100, H = NP$
Rep-CLPSO	$NP = 100, m = 7, c = 2.0$
jDE-WeB	$NP = 100, \tau_1 = \tau_2 = 0.1$
JADE-WeB	$NP = 100, c = 0.1, \mu_{CR} = 0.5, \mu_F = 0.5$

S-III. Supplemental Results

TABLE S-R-II
COMPARISON OF DIFFERENT METHODS ON TEST INSTANCES F01-F21 WITH RESPECT TO THE PEAK RATIO. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	A-MONES	A-MOMMOP	A-MOBiDE	NCDE	NSDE	LIPS	R3PSO	Rep-SHADE	Rep-CLPSO
F01	**1.0000**	**1.0000**	**1.0000**	0.7100	**1.0000**	**1.0000**	**1.0000**	0.0700	**1.0000**	**1.0000**
F02	0.6200	0.5500	0.0500	0.0000	**0.8300**	0.3400	0.0000	0.0000	0.0000	0.0000
F03	**1.0000**	**1.0000**	**1.0000**	0.1345	0.9873	0.9600	0.6382	0.1309	0.9873	0.9455
F04	0.9573	0.7387	0.9000	0.1560	0.9773	0.9653	0.4813	0.1213	0.9147	**0.9800**
F05	**1.0000**	0.9708	0.5092	0.0985	0.6400	0.8138	0.0923	0.0000	0.7754	0.5015
F06	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	0.0000	**1.0000**	**1.0000**
F07	0.9400	0.5625	0.9725	0.2075	0.9350	0.9650	0.1925	0.0200	**0.9975**	0.9675
F08	**0.4200**	0.4000	0.1600	0.0200	0.1000	0.0400	0.0000	0.0000	0.0000	0.0000
F09	0.8371	0.6029	0.7429	0.2086	**0.9257**	0.8943	0.2600	0.1657	0.8514	0.8257
F10	**0.8933**	0.7333	0.7867	0.0000	0.0000	0.0733	0.0000	0.0000	0.2933	0.0000
F11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.0000	0.0200	**1.0000**	**1.0000**
F12	0.8880	0.7180	0.8840	0.1860	0.6540	0.8680	0.1340	0.0140	**0.9240**	0.8400
F13	0.9733	**0.9956**	0.9889	0.2178	0.9800	0.9867	0.0756	0.0000	0.9778	0.8667
F14	**1.0000**	0.4431	0.9985	0.2800	0.8508	0.8508	0.1246	0.0031	0.8877	0.9277
F15	0.6688	0.1738	**0.9138**	0.1200	0.7700	0.7388	0.0000	0.0000	0.6925	0.6000
F16	0.9433	0.7567	0.8600	0.3600	**1.0000**	**1.0000**	0.2267	0.0100	**1.0000**	**1.0000**
F17	0.6200	0.3200	0.9000	0.0000	0.2800	0.2100	0.0000	0.0000	**1.0000**	0.6100
F18	**0.9514**	0.5686	0.5829	0.2171	0.8371	0.9457	0.0029	0.0029	0.9286	0.7629
F19	0.9950	0.5100	0.5700	0.4250	**1.0000**	**1.0000**	0.2600	0.0100	**1.0000**	**1.0000**
F20	**1.0000**	0.7633	**1.0000**	0.2967	0.9967	0.9900	0.1767	0.0033	**1.0000**	**1.0000**
F21	0.8550	0.6250	0.8250	0.3150	**0.9650**	0.9600	0.0000	0.0100	0.7450	0.4850
Average	**0.8839**	0.6873	0.7926	0.2835	0.7966	0.7906	0.1745	0.0277	0.8083	0.7292

TABLE S-R-III
COMPARISON OF DIFFERENT METHODS ON TEST CASES F01-F21 WITH RESPECT TO THE SUCCESS RATE. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	A-MONES	A-MOMMOP	A-MOBiDE	NCDE	NSDE	LIPS	R3PSO	Rep-SHADE	Rep-CLPSO
F01	**1.00**	**1.00**	**1.00**	0.42	**1.00**	**1.00**	**1.00**	0.02	**1.00**	**1.00**
F02	0.36	0.10	0.00	0.00	**0.68**	0.08	0.00	0.00	0.00	0.00
F03	**1.00**	**1.00**	**1.00**	0.00	0.88	0.66	0.00	0.00	0.86	0.62
F04	0.58	0.36	0.38	0.00	**0.72**	0.62	0.00	0.00	0.24	**0.72**
F05	**1.00**	0.86	0.00	0.00	0.00	0.04	0.00	0.00	0.08	0.00
F06	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.00	0.00	**1.00**	**1.00**
F07	0.60	0.50	0.80	0.00	0.52	0.72	0.00	0.00	**0.98**	0.78
F08	**0.42**	0.40	0.16	0.02	0.10	0.04	0.00	0.00	0.00	0.00
F09	0.12	0.02	0.00	0.02	**0.50**	0.32	0.00	0.00	0.06	0.02
F10	**0.68**	0.50	0.42	0.00	0.00	0.00	0.00	0.00	0.04	0.00
F11	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	0.00	0.02	**1.00**	**1.00**
F12	0.28	**0.50**	0.24	0.00	0.00	0.28	0.00	0.00	0.42	0.22
F13	0.76	**0.96**	0.90	0.00	0.84	0.88	0.00	0.00	0.80	0.28
F14	**1.00**	0.00	0.98	0.00	0.04	0.02	0.00	0.00	0.10	0.36
F15	0.00	0.00	**0.36**	0.00	0.02	0.00	0.00	0.00	0.00	0.00
F16	0.66	0.50	0.48	0.00	**1.00**	**1.00**	0.00	0.00	**1.00**	**1.00**
F17	0.24	0.12	0.80	0.00	0.00	0.00	0.00	0.00	**1.00**	0.28
F18	**0.70**	0.00	0.00	0.00	0.04	0.64	0.00	0.00	0.62	0.12
F19	0.98	0.00	0.14	0.02	**1.00**	**1.00**	0.00	0.00	**1.00**	**1.00**
F20	**1.00**	0.50	**1.00**	0.00	0.98	0.94	0.00	0.00	**1.00**	**1.00**
F21	0.14	0.00	0.14	0.00	**0.74**	**0.74**	0.00	0.00	0.08	0.00
Average	**0.64**	0.44	0.51	0.12	0.53	0.52	0.05	0.00	0.54	0.45

TABLE S-R-IV
COMPARISON OF DIFFERENT METHODS ON TEST CASES F26-F38 WITH RESPECT TO THE PEAK RATIO. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	A-MONES	A-MOMMOP	NCDE	NSDE	Rep-SHADE	Rep-CLPSO
F26	**1.0000**	0.9700	0.9800	0.9900	0.9900	0.9800	0.8100
F27	0.0933	0.3750	0.0000	0.6467	**0.7033**	0.2733	0.4533
F28	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	**1.0000**	0.7550	**1.0000**	0.9700	**1.0000**	**1.0000**	0.9900
F31	**1.0000**	0.8600	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9900
F32	**1.0000**	**1.0000**	**1.0000**	0.4000	0.8400	**1.0000**	**1.0000**
F33	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9800	**1.0000**	**1.0000**
F34	0.8800	**0.9200**	0.5200	0.1600	0.6600	0.4200	0.1400
F35	**1.0000**	0.9467	0.9933	0.9800	**1.0000**	**1.0000**	**1.0000**
F36	0.9400	0.9100	0.9400	**1.0000**	**1.0000**	**1.0000**	0.9200
F37	0.9320	0.9160	0.9000	0.7880	**0.9840**	0.9480	0.3800
F38	**1.0000**	0.7600	**1.0000**	0.9750	**1.0000**	**1.0000**	**1.0000**
Average	0.9112	0.8779	0.8718	0.8392	**0.9352**	0.8939	0.8218

TABLE S-R-V

COMPARISON OF DIFFERENT METHODS ON TEST CASES F26-F38 WITH RESPECT TO THE SUCCESS RATE. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	A-MONES	A-MOMMOP	NCDE	NSDE	Rep-SHADE	Rep-CLPSO
F26	**1.00**	0.94	0.96	0.98	0.98	0.96	0.62
F27	0.00	0.00	0.00	0.00	**0.02**	0.00	0.00
F28	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F29	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F30	**1.00**	0.50	**1.00**	0.88	**1.00**	**1.00**	0.96
F31	**1.00**	0.74	**1.00**	**1.00**	**1.00**	**1.00**	0.98
F32	**1.00**	**1.00**	**1.00**	0.40	0.84	**1.00**	**1.00**
F33	**1.00**	**1.00**	**1.00**	**1.00**	0.96	**1.00**	**1.00**
F34	0.88	**0.96**	0.52	0.16	0.66	0.42	0.14
F35	**1.00**	0.86	0.98	0.96	**1.00**	**1.00**	**1.00**
F36	0.88	0.84	0.88	**1.00**	**1.00**	**1.00**	0.84
F37	0.66	0.58	0.54	0.18	**0.92**	0.74	0.00
F38	**1.00**	0.50	**1.00**	0.90	**1.00**	**1.00**	**1.00**
Average	**0.88**	0.76	0.84	0.73	**0.88**	0.86	0.73

TABLE S-R-VI

INFLUENCE OF THE HISTORICAL MEMORY SIZE (H) ON THE PERFORMANCE OF A-WEB FOR TEST INSTANCES F01-F21 WITH RESPECT TO THE PEAK RATIO. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	$H=5$	$H=10$	$H=30$	$H=50$	$H=100$	$H=200$	$H=300$	$H=400$	$H=500$
F01	**1.0000**	**1.0000**	**1.0000**	0.9900	**1.0000**	**1.0000**	**1.0000**	0.9900	**1.0000**
F02	0.0100	0.0200	0.2400	0.4700	0.6200	0.6900	0.6700	0.7000	**0.7300**
F03	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F04	0.9413	0.9427	**0.9680**	0.9613	0.9573	0.9533	0.9667	0.9600	0.9627
F05	0.9969	0.9969	0.9969	0.9985	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F06	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F07	0.8875	0.8775	0.9250	0.9550	0.9400	**0.9800**	0.9675	0.9575	0.9450
F08	0.3200	0.4200	0.4000	0.4800	0.4200	0.4600	0.4200	**0.5600**	0.4800
F09	0.7886	0.7771	0.8257	0.8200	0.8371	**0.8429**	0.8086	0.8400	0.8171
F10	0.3067	0.4333	0.7533	0.8533	0.8933	0.9467	**0.9733**	**0.9733**	0.9533
F11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F12	0.8220	0.8380	0.8860	0.8700	0.8880	0.8940	**0.9040**	0.8880	0.8720
F13	**0.9933**	**0.9933**	0.9844	0.9756	0.9733	0.9756	0.9600	0.9867	0.9689
F14	**1.0000**	0.9954	0.9985	0.9985	**1.0000**	**1.0000**	**1.0000**	0.9969	**1.0000**
F15	0.5838	0.6388	0.6875	0.7075	0.6688	0.6938	0.7088	0.7013	**0.7225**
F16	0.9567	0.9567	0.9600	0.9733	0.9433	0.9567	0.9667	0.9733	**0.9767**
F17	0.5900	**0.6200**	0.5800	0.5800	**0.6200**	0.5500	0.5200	0.4900	0.4900
F18	0.8886	0.9200	**0.9714**	0.9543	0.9514	0.9514	0.9343	0.9343	0.9400
F19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9950	**1.0000**	**1.0000**	**1.0000**	0.9950
F20	0.9933	0.9867	**1.0000**	**1.0000**	**1.0000**	0.9967	**1.0000**	0.9967	**1.0000**
F21	**0.8650**	0.8600	0.8225	0.8650	0.8550	0.8575	0.8600	0.8625	0.8400
Average	0.8068	0.8227	0.8571	0.8787	0.8839	0.8928	0.8886	**0.8957**	0.8902

TABLE S-R-VII

INFLUENCE OF THE HISTORICAL MEMORY SIZE (H) ON THE PERFORMANCE OF A-WEB FOR TEST INSTANCES F01-F21 WITH RESPECT TO THE SUCCESS RATE. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	$H=5$	$H=10$	$H=30$	$H=50$	$H=100$	$H=200$	$H=300$	$H=400$	$H=500$
F01	**1.00**	**1.00**	**1.00**	0.98	**1.00**	**1.00**	**1.00**	0.98	**1.00**
F02	0.00	0.00	0.12	0.22	0.36	0.44	0.40	0.44	**0.50**
F03	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F04	0.44	0.40	0.60	0.52	0.58	0.52	0.60	0.56	**0.68**
F05	0.96	0.96	0.96	0.98	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F06	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F07	0.44	0.38	0.48	0.68	0.60	**0.84**	0.74	0.68	0.60
F08	0.32	0.42	0.40	0.48	0.42	0.46	0.42	**0.56**	0.48
F09	0.02	0.00	0.10	0.06	**0.12**	0.08	0.02	**0.12**	0.04
F10	0.02	0.10	0.28	0.58	0.68	0.84	**0.92**	**0.92**	0.86
F11	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F12	0.12	0.14	0.24	0.26	0.28	0.36	**0.40**	0.36	0.26
F13	**0.94**	**0.94**	0.86	0.78	0.76	0.78	0.66	0.88	0.72
F14	**1.00**	0.94	0.98	0.98	**1.00**	**1.00**	**1.00**	0.96	**1.00**
F15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F16	0.74	0.74	0.76	0.84	0.66	0.74	0.80	0.84	**0.86**
F17	0.18	**0.24**	0.16	0.16	**0.24**	0.10	0.08	0.08	0.08
F18	0.38	0.52	**0.80**	0.72	0.70	0.70	0.60	0.62	0.62
F19	**1.00**	**1.00**	**1.00**	**1.00**	0.98	**1.00**	**1.00**	**1.00**	0.98
F20	0.96	0.92	**1.00**	**1.00**	**1.00**	0.98	**1.00**	0.98	**1.00**
F21	0.28	0.32	0.20	0.22	0.14	0.22	0.20	**0.36**	0.24
Average	0.56	0.57	0.62	0.64	0.64	0.67	0.66	**0.68**	0.66

TABLE S-R-VIII

INFLUENCE OF F AND CR ON THE PERFORMANCE OF A-WEB FOR TEST INSTANCES F01-F21 WITH RESPECT TO THE PEAK RATIO. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	A-WeB-2	A-WeB-3	A-WeB-4	A-WeB-5
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**0.6200**	0.1700	0.0000	0.5000	0.0300
F03	**1.0000**	0.9964	**1.0000**	**1.0000**	**1.0000**
F04	0.9573	0.9013	0.9720	0.9280	**0.9880**
F05	**1.0000**	0.5708	0.9846	**1.0000**	**1.0000**
F06	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**
F07	**0.9400**	0.8650	0.6750	0.8250	0.8225
F08	0.4200	0.1000	0.0200	0.4200	**0.9600**
F09	**0.8371**	0.8229	0.8343	0.8286	0.8229
F10	0.8933	0.0333	0.8267	0.8600	**0.9000**
F11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F12	0.8880	0.8220	**0.9260**	0.8720	0.9080
F13	**0.9733**	0.7489	0.7000	0.9489	0.8756
F14	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F15	0.6688	0.5575	0.0350	**0.6838**	0.5513
F16	0.9433	**0.9567**	0.9200	0.9500	0.9233
F17	**0.6200**	0.0000	0.0000	0.0200	0.0400
F18	**0.9514**	0.7829	0.8886	0.8371	0.8857
F19	0.9950	**1.0000**	**1.0000**	0.9950	**1.0000**
F20	**1.0000**	0.9833	**1.0000**	0.9900	**1.0000**
F21	**0.8550**	0.7150	0.7475	0.8425	0.7325
Average	**0.8839**	0.7155	0.6919	0.8334	0.8305

TABLE S-R-IX

INFLUENCE OF F AND CR ON THE PERFORMANCE OF A-WEB FOR TEST INSTANCES F01-F21 WITH RESPECT TO THE SUCCESS RATE. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	A-WeB-2	A-WeB-3	A-WeB-4	A-WeB-5
F01	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F02	**0.36**	0.06	0.00	0.12	0.00
F03	**1.00**	0.96	**1.00**	**1.00**	**1.00**
F04	0.58	0.26	0.68	0.30	**0.82**
F05	**1.00**	0.00	0.86	**1.00**	**1.00**
F06	**1.00**	**1.00**	0.00	**1.00**	**1.00**
F07	**0.60**	0.00	0.26	0.20	0.18
F08	0.42	0.10	0.02	0.42	**0.96**
F09	0.12	0.06	**0.14**	0.08	0.12
F10	0.68	0.00	0.52	0.62	**0.70**
F11	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F12	0.28	0.08	**0.44**	0.22	0.32
F13	**0.76**	0.04	0.02	0.60	0.16
F14	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
F15	0.00	0.00	0.00	0.00	0.00
F16	0.66	**0.76**	0.52	0.70	0.58
F17	**0.24**	0.00	0.00	0.00	0.00
F18	**0.70**	0.10	0.22	0.12	0.30
F19	0.98	**1.00**	**1.00**	0.98	**1.00**
F20	**1.00**	0.90	**1.00**	0.94	**1.00**
F21	0.14	0.00	0.04	**0.18**	0.06
Average	**0.64**	0.40	0.46	0.55	0.58

TABLE S-R-X

INFLUENCE OF THE PARAMETER ADAPTATION ON THE PERFORMANCE OF A-WEB FOR TEST INSTANCES F01-F21 WITH RESPECT TO THE PEAK RATIO. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	jDE-WeB	JADE-WeB
F01	**1.0000**	**1.0000**	**1.0000**
F02	0.6200	0.3600	**0.7000**
F03	**1.0000**	**1.0000**	**1.0000**
F04	0.9573	0.9467	**0.9653**
F05	**1.0000**	0.9969	**1.0000**
F06	**1.0000**	**1.0000**	**1.0000**
F07	0.9400	0.8700	**0.9725**
F08	0.4200	**0.7200**	0.1800
F09	0.8371	0.8086	**0.8514**
F10	0.8933	**0.9467**	0.2467
F11	**1.0000**	**1.0000**	**1.0000**
F12	0.8880	0.8240	**0.9080**
F13	**0.9733**	0.9667	0.9444
F14	**1.0000**	**1.0000**	**1.0000**
F15	0.6688	0.4938	**0.7363**
F16	0.9433	0.9467	**0.9600**
F17	**0.6200**	0.5900	0.2900
F18	**0.9514**	0.8600	0.8629
F19	0.9950	0.9950	**1.0000**
F20	**1.0000**	**1.0000**	**1.0000**
F21	0.8550	0.7650	**0.8575**
Average	**0.8839**	0.8614	0.8321

TABLE S-R-XI

INFLUENCE OF THE PARAMETER ADAPTATION ON THE PERFORMANCE OF A-WEB FOR TEST INSTANCES F01-F21 WITH RESPECT TO THE SUCCESS RATE. THE BEST RESULT FOR EACH TEST INSTANCE AMONG THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**.

Instance	A-WeB	jDE-WeB	JADE-WeB
F01	**1.00**	**1.00**	**1.00**
F02	0.36	0.10	**0.46**
F03	**1.00**	**1.00**	**1.00**
F04	0.58	0.48	**0.60**
F05	**1.00**	0.96	**1.00**
F06	**1.00**	**1.00**	**1.00**
F07	0.60	0.28	**0.86**
F08	0.42	**0.72**	0.18
F09	**0.12**	0.04	**0.12**
F10	0.68	**0.84**	0.06
F11	**1.00**	**1.00**	**1.00**
F12	0.28	0.14	**0.42**
F13	**0.76**	0.74	0.52
F14	**1.00**	**1.00**	**1.00**
F15	0.00	0.00	**0.02**
F16	0.66	0.68	**0.76**
F17	**0.24**	0.04	0.04
F18	**0.70**	0.22	0.24
F19	0.98	0.98	**1.00**
F20	**1.00**	**1.00**	**1.00**
F21	0.14	0.04	**0.24**
Average	**0.64**	0.58	0.60

TABLE S-R-XII

COMPARISON BETWEEN "DE/CURRENT/1" AND "DE/RAND/1" IN A-WEB. THE BETTER RESULT FOR EACH TEST INSTANCE BETWEEN THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**. IN THE LAST ROW, THE RESULTS IN THE FORM OF (R^+, R^-, p) ARE OBTAINED BY THE MULTIPLE-PROBLEM WILCOXON TEST.

Instance	PR		SR	
	DE/current/1	DE/rand/1	DE/current/1	DE/rand/1
F02	0.6200	**0.9100**	0.36	**0.82**
F04	0.9573	**0.9760**	0.58	**0.70**
F05	**1.0000**	0.9662	**1.00**	0.66
F07	**0.9400**	0.9075	**0.60**	0.52
F08	**0.4200**	0.2400	**0.42**	0.24
F09	**0.8371**	0.8114	**0.12**	0.08
F10	**0.8933**	0.7133	**0.68**	0.14
F12	0.8880	**0.9060**	0.28	**0.34**
F13	**0.9733**	0.7333	**0.76**	0.04
F15	**0.6688**	0.6438	0.00	0.00
F16	**0.9433**	0.8433	**0.66**	0.30
F17	0.6200	**0.6800**	0.24	**0.36**
F18	**0.9514**	0.9171	**0.70**	0.48
F19	0.9950	**1.0000**	0.98	**1.00**
F21	**0.8550**	0.6375	**0.14**	0.04
Average	**0.8375**	0.7924	**0.50**	0.38
Wilcoxon test	(90.0, 30.0, 9.46E-02)		(83.5, 36.5, 1.53E-01)	

TABLE S-R-XIII
INFLUENCE OF THE DISTANCE COMPARISON CRITERION FOR A-WEB. THE BETTER RESULT FOR EACH TEST INSTANCE BETWEEN THE COMPARED METHODS IS HIGHLIGHTED IN **BOLDFACE**. IN THE LAST ROW, THE RESULTS IN THE FORM OF (R^{+}, R^{-}, p) ARE OBTAINED BY THE MULTIPLE-PROBLEM WILCOXON TEST.

Instance	PR		SR	
	A-WeB	A-WeB-6	A-WeB	A-WeB-6
F02	0.6200	**0.6300**	0.36	**0.46**
F04	0.9573	**0.9680**	0.58	**0.66**
F05	**1.0000**	0.7354	**1.00**	0.10
F07	0.9400	**0.9925**	0.60	**0.94**
F08	**0.4200**	0.2800	**0.42**	0.28
F09	0.8371	**0.8743**	0.12	**0.18**
F10	0.8933	**0.9800**	0.68	**0.94**
F12	0.8880	**0.9300**	0.28	**0.56**
F13	**0.9733**	0.9711	**0.76**	0.74
F14	**1.0000**	0.9985	**1.00**	0.98
F15	0.6688	**0.7713**	0.00	0.00
F16	0.9433	**0.9867**	0.66	**0.92**
F17	0.6200	**0.7000**	0.24	**0.40**
F18	0.9514	**0.9771**	0.70	**0.84**
F19	0.9950	**1.0000**	0.98	**1.00**
F21	0.8550	**0.9800**	0.14	**0.86**
Average	0.8477	**0.8609**	0.53	**0.62**
Wilcoxon test	(34.0, 102.0, $\geq$ 0.2)		(34.0, 102.0, $\geq$ 0.2)	