

**Supplementary File of “A Distance-based Locally Informed  
Particle Swarm Model for Multi-modal Optimization”**

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## Test Function F1-F15

F1: Equal Maxima

$$f_1(x) = \sin^6(5\pi x)$$

Range:  $0 \leq x \leq 1$

F2: Decreasing Maxima

$$f_2(x) = \exp[-2\log(2) \cdot (\frac{x-0.1}{0.8})^2] \cdot \sin^6(5\pi x)$$

Range:  $0 \leq x \leq 1$

F3: Uneven Maxima

$$f_3(x) = \sin^6(5\pi(x^{3/4} - 0.05))$$

Range:  $0 \leq x \leq 1$

F4: Uneven Decreasing Maxima

$$f_4(x) = \exp[-2\log(2) \cdot (\frac{x-0.08}{0.854})^2] \cdot \sin^6(5\pi(x^{3/4} - 0.05))$$

Range:  $0 \leq x \leq 1$

F5: Himmelblau's function

$$f_5(x, y) = 200 - (x^2 + y - 11)^2 - (x + y^2 - 7)^2$$

Range:  $-6 \leq x, y \leq 6$

F6: Six-Hump Camel Back

$$f_6(x, y) = -4[(4 - 2.1x^2 + \frac{x^4}{3})x^2 + xy + (-4 + 4y^2)y^2]$$

Range:  $-1.9 \leq x \leq 1.9;$   
 $-1.1 \leq y \leq 1.1$

F7: Shekel's foxholes

$$f_7(x, y) = 500 - \frac{1}{0.002 + \sum_{i=0}^{24} \frac{1}{1+i+(x-a(i))^6+(y-b(i))^6}}$$

where  $a(i) = 16(i \bmod 5) - 2$ , and  $b(i) = 16\lfloor (i/5) \rfloor - 2$

Range:  $-65.536 \leq x, y \leq 65.535$

F8: 2D Inverted Shubert function

$$f_8(\vec{x}) = -\prod_{i=1}^2 \sum_{j=1}^5 j \cos[(j+1)x_i + j]$$

Range:  $-10 \leq x_1, x_2 \leq 10$

F9: Waves

$$f_9(x, y) = (0.3x)^3 - (y^2 - 4.5y^2)xy - 4.7 \cos(3x - y^2(2+x)) \sin(2.5\pi x)$$

Range:  $-0.9 \leq x \leq 1.2, -1.2 \leq y \leq 1.2$

F10: Sphere

$$f_{10}(x) = \sum_{i=1}^D (-x_i)^2$$

Range:  $-5.12 \leq x_i \leq 5.12$

F11: Branin RCOS

$$f_{11}(x, y) = (y - \frac{5.1}{4\pi^2}x^2 + \frac{5}{\pi}x - 6)^2 + 10(1 - \frac{1}{8\pi})\cos(x) + 10$$

Range:  $-5 \leq x_i \leq 10, 0 \leq y \leq 15$

F12: Ackley

$$f_{12}(x, y) = 20 + e - 20e^{-0.2\sqrt{\frac{x^2+y^2}{2}}} - e^{\frac{\cos(2\pi x) + \cos(2\pi y)}{2}}$$

Range:  $-30 \leq x, y \leq 30$

F13: Michalewicz

$$f_{13}(x, y) = \sin(x) \sin^{20}\left(\frac{x^2}{\pi}\right) + \sin(y) \sin^{20}\left(\frac{2y^2}{\pi}\right)$$

Range:  $0 \leq x, y \leq \pi$

F14: Ursem F1

$$f_{11}(x, y) = \sin(2x - 0.5\pi) + 3\cos(y) + 0.5x$$

Range:  $-2.5 \leq x \leq 3, -2 \leq y \leq 2$

F15: Ursem F3

$$f_{15}(x, y) = \sin(2.2\pi x + 0.5\pi) \cdot \frac{2-|y|}{2} \cdot \frac{3-|x|}{2} + \sin(0.5\pi y^2 + 0.5\pi) \cdot \frac{2-|y|}{2} \cdot \frac{2-|x|}{2}$$

Range:  $-2.5 \leq x \leq 3, -2 \leq y \leq 2$

### Test Function F16-F30

These 15 composition functions are defined as follow:

$F(x)$  : new composition function

$f_i(x)$  :  $i^{\text{th}}$  basic function used to construct the composition function.

$n$  : number of basic functions (number of optima)

$D$  : dimensions (can be chosen from 1-100)

$M_i$  : linear transformation matrix for each  $f_i(x)$

$o_i$  : new shifted optima position for each  $f_i(x)$

$$F(x) = \sum_{i=1}^n \{w_i * [f_i((x - o_i) / \lambda_i * M_i)]\}$$

$w_i$  : weight value for each  $f_i(x)$ , calculated as follow:

$$w_i = \exp\left(-\frac{\sum_{k=1}^D (x_k - o_{ik})}{2D\sigma_i^2}\right)$$

$$w_i = \begin{cases} w_i & w_i == \max(w_i) \\ w_i * (1 - \max(w_i))^10 & w_i \neq \max(w_i) \end{cases}$$

Then normalize the weight  $w_i = w_i / \sum_{i=1}^n w_i$

$\sigma_i$  : used to control each  $f_i(x)$ 's coverage range.

$\lambda_i$  : used to stretch compress the function.

$f_i'(x) = C * f_i(x) / |f_{\max i}|$ ,  $C$  is a predefined constant.

$|f_{\max i}|$  is estimated using:  $|f_{\max i}| = f_i((x' / \lambda_i) * M_i), x' = [5, 5, \dots, 5]$

### Composition Function 1 (F16, $n=8$ )

$f_{1-2}(x)$  : Rastrigin's Function

$$f_i(x) = \sum_{i=1}^D (x_i^2 - 10 \cos(2\pi x_i) + 10)$$

$f_{3-4}(x)$  : Weierstrass Function

$$f_i(x) = \sum_{i=1}^D \left( \sum_{k=0}^{k_{\max}} [a^k \cos(2\pi b^k (x_i + 0.5))] \right) - \left( \sum_{k=0}^{k_{\max}} D \sum_{k=0}^{k_{\max}} [a^k \cos(2\pi b^k \cdot 0.5)] \right)$$

$$a = 0.5, b = 3, k_{\max} = 20$$

$f_{5-6}(x)$  : Griewank's Function

$$f_i(x) = \sum_{i=1}^D \frac{x_i^2}{4000} - \prod_{i=1}^D \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

$f_{7-8}(x)$  : Sphere Function

$$f_i(x) = \sum_{i=1}^D x_i^2$$

$\sigma_i = 1$  for all  $i$

$$\lambda = [1, 1, 10, 10, 5/60, 5/60, 5/32, 5/32]$$

$M_i$  : are all identity matrices

These formulas are basic functions; shift and rotation should be added to these functions. Take  $f_1$  as an example, the following function should be evaluated:

$$f_i(z) = \sum_{i=1}^D (z_i^2 - 10 \cos(2\pi z_i) + 10)$$

where  $z = ((x - o_i) / \lambda_i) * M_1$ .

### Composition Function 2 (F17 $n=6$ )

$f_{1-2}(x)$  : Griewank's Function

$f_{3-4}(x)$  : Weierstrass Function

$f_{5-6}(x)$  : Sphere Function

$\sigma_i = 1$  for all  $i$

$\lambda = [1, 1, 10, 10, 5/60, 5/60,]$

$M_i$  : are all identity matrices

### Composition Function 3 (F18 $n=6$ )

$f_{1-2}(x)$  : Rastrigin's Function

$f_{3-4}(x)$  : Griewank's Function

$f_{5-6}(x)$  : Sphere Function

$\sigma_i = 1$  for all  $i$

$\lambda = [1, 1, 10, 10, 5/60, 5/60,]$

$M_i$  : are all identity matrices

### Composition Function 4 (F19 $n=6$ )

$f_{1-2}(x)$  : Rastrigin's Function

$f_{3-4}(x)$  : Weierstrass Function

$f_{5-6}(x)$  : Griewank's Function

$\sigma_i = 1$  for all  $i$

$\lambda = [1, 1, 10, 10, 5/60, 5/60,]$

$M_i$  : are all identity matrices

### Composition Function 5 (F20 $n=6$ )

$f_{1-2}(x)$  : Rastrigin's Function

$f_{3-4}(x)$  : Weierstrass Function

$f_{5-6}(x)$  : Sphere Function

$\sigma_i = 1$  for all  $i$

$\lambda = [1, 1, 10, 10, 5/60, 5/60,]$

$M_i$  : are all identity matrices

### Composition Function 6 (F21 $n=6$ )

$f_{1-2}(x)$  : F8F2 Function

$$F8(x) = \sum_{i=1}^D \frac{x_i^2}{4000} - \prod_{i=1}^D \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

$$F2(x) = \sum_{i=1}^{D-1} (100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2)$$

$$f_i(x) = F8(F2(x_1, x_2)) + F8(F2(x_2, x_3)) + \dots + F8(F2(x_{D-1}, x_D)) + F8(F2(x_D, x_1))$$

$f_{3-4}(x)$  : Weierstrass Function

$f_{5-6}(x)$  : Griewank's Function

$\sigma = [1, 1, 1, 1, 1, 2],$

$\lambda = [5 * 5 / 100; 5 / 100; 5 * 1; 1; 5 * 1; 1]$

$M_i$  : are all orthogonal matrix

### Composition Function 7 (F22 $n=6$ )

$$f_{1-2}(x) : \text{Rotated Expanded Scaffer's F6 Function} \quad F(x, y) = 0.5 + \frac{(\sin^2(\sqrt{x^2 + y^2}) - 0.5)}{(1 + 0.001(x^2 + y^2))^2}$$

$$f_i(x) = F(x_1, x_2) + F(x_2, x_3) + \dots + F(x_{D-1}, x_D) + F(x_D, x_1)$$

$$f_{3-4}(x) : \text{F8F2 Function}$$

$$f_{5-6}(x) : \text{Weierstrass Function}$$

$$\sigma = [1, 1, 1, 1, 1, 2],$$

$$\lambda = [5; 10; 5; 1; 5 * 5 / 100; 5 / 100]$$

$M_i$  : are all orthogonal matrix

### Composition Function 8 (F23 $n=6$ )

$$f_{1-2}(x) : \text{Rotated Expanded Scaffer's F6 Function}$$

$$f_{3-4}(x) : \text{F8F2 Function}$$

$$f_{5-6}(x) : \text{Griewank's Function}$$

$$\sigma = [1, 1, 1, 1, 1, 2],$$

$$\lambda = [5 * 5 / 100; 5 / 100; 5 * 1; 1; 5 * 1; 1]$$

$M_i$  : are all orthogonal matrix

### Composition Function 9 (F24 $n=6$ )

$$f_{1-2}(x) : \text{Rotated Expanded Scaffer's F6 Function}$$

$$f_{3-4}(x) : \text{Weierstrass Function}$$

$$f_{5-6}(x) : \text{Griewank's Function}$$

$$\sigma = [1, 1, 1, 1, 1, 2],$$

$$\lambda = [5; 10; 5 * 5 / 100; 5 / 100; 5; 1]$$

$M_i$  : are all orthogonal matrix

### **Composition Function 10 (F25 $n=6$ )**

$f_{1-2}(x)$  : Rastrigin's Function

$f_{3-4}(x)$  : F8F2 Function

$f_{5-6}(x)$  : Weierstrass Function

$$\sigma = [1, 1, 1, 1, 1, 2],$$

$$\lambda = [5; 10; 5 * 5 / 100; 5 / 100; 5; 1]$$

$M_i$  : are all orthogonal matrix

### **Composition Function 11 (F26 $n=8$ )**

$f_{1-2}(x)$  : Rastrigin's Function

$f_{3-4}(x)$  : F8F2 Function

$f_{5-6}(x)$  : Weierstrass Function

$f_{7-8}(x)$  : Griewank's Function

$$\sigma = [1, 1, 1, 1, 1, 2, 2, 2],$$

$$\lambda = [5; 1; 5; 1; 50; 10; 5 * 5 / 200; 5 / 200]$$

$M_i$  : are all orthogonal matrix

### **Composition Function 12 (F27 $n=8$ )**

$f_{1-2}(x)$  : Rotated Expanded Scaffer's F6 Function

$f_{3-4}(x)$  : F8F2 Function

$f_{5-6}(x)$  : Weierstrass Function

$f_{7-8}(x)$  : Griewank's Function

$\sigma = [1, 1, 1, 1, 1, 2, 2, 2]$ ,

$\lambda = [5 * 5 / 100; 5 / 100; 5; 1; 5; 1; 50; 10]$

$M_i$  : are all orthogonal matrix

### **Composition Function 13 (F28 $n=10$ )**

$f_{1-2}(x)$  : Rotated Expanded Scaffer's F6 Function

$f_{3-4}(x)$  : Rastrigin's Function

$f_{5-6}(x)$  : F8F2 Function

$f_{7-8}(x)$  : Weierstrass Function

$f_{9-10}(x)$  : Griewank's Function

$\sigma = [1, 1, 1, 1, 1, 2, 2, 2, 2, 2]$ ,

$\lambda = [5 * 5 / 100; 5 / 100; 5; 1; 5; 1; 50; 10; 5 * 5 / 200; 5 / 200]$

$M_i$  : are all orthogonal matrix

### **Composition Function 14 (F29 $n=10$ )**

All settings are the same as F13, except  $M_i$ 's condition numbers are [10 20 50 100 200 1000 2000 3000 4000 5000]

### **Composition Function 15 (F30 $n=10$ )**

$f_1(x)$  : Weierstrass Function

$f_2(x)$  : Rotated Expanded Scaffer's F6 Function

$f_3(x)$  : F8F2 Function

$f_4(x)$  : Ackley's Function

$$f_i(x) = -20 \exp(-0.2 \sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2}) - \exp(\frac{1}{D} \sum_{i=1}^D \cos(2\pi x_i)) + 20 + e \quad f_5(x) : \text{Rastrigin's Function}$$

$f_6(x)$  : Griewank's Function

$f_7(x)$  : Non-Continuous Expanded Scaffer's F6 Function

$$F(x, y) = 0.5 + \frac{(\sin^2(\sqrt{x^2 + y^2}) - 0.5)}{(1 + 0.001(x^2 + y^2))^2}$$

$$f_i(x) = F(y_1, y_2) + F(y_2, y_3) + \dots + F(y_{D-1}, y_D) + F(y_D, y_1)$$

$$y_i = \begin{cases} x_j & |x_j| < 1/2 \\ \text{round}(2x_j)/2 & |x_j| > 1/2 \end{cases} \text{ for } j = 1, 2, \dots, D$$

$$\text{round}(x) = \begin{cases} a-1 & \text{if } x \leq 0 \text{ \& } b \geq 0.5 \\ a & \text{if } b < 0.5 \\ a+1 & \text{if } x > 0 \text{ \& } b \geq 0.5 \end{cases}$$

$f_8(x)$  : Non-Continuous Rastrigin's Function

$$f_i(x) = \sum_{i=1}^D (y_i^2 - 10 \cos(2\pi y_i) + 10)$$

$$y_i = \begin{cases} x_j & |x_j| < 1/2 \\ \text{round}(2x_j)/2 & |x_j| > 1/2 \end{cases} \text{ for } j = 1, 2, \dots, D$$

$f_9(x)$  : High Conditioned Elliptic Function

$$f(x) = \sum_{i=1}^D (10^6)^{\frac{i-1}{D-1}} x_i^2$$

$f_{10}(x)$  : Sphere Function with Noise in Fitness

$$f_i(x) = (\sum_{i=1}^D x_i^2) (1 + 0.1 |N(0, 1)|)$$

$n=10$

$\sigma_i = 2$  for all  $i$

$$\lambda = [10; 5/20; 1; 5/32; 1; 5/100; 5/50; 1; 5/100; 5/100]$$

$M_i$  are all rotation matrices, condition number are [100 50 30 10 5 5 4 3 2 2];

Table VIII. Peak ratio (test functions F1 – F15)

Test Func.		LIPS	$r_{2pso}$	$r_{3pso}$	$r_{2pso-lhc}$	$r_{3pso-lhc}$	SPSO	FERPSO	SDE	CDE	SACMA-ES
F1	Worst	1.00	0.80	0.80	1.00	0.80	0.80	0.80	0.80	0.20	0.00
	Best	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.20
	Mean	<b>1.00</b>	0.98	0.98	<b>1.00</b>	0.98	0.98	0.97	0.94	0.77	0.01
	Std	0.00	0.06	0.07	0.00	0.06	0.07	0.07	0.09	0.21	0.04
	$t$ -test	-	0	0	0	0	0	0	1	1	1
F2	Worst	0.80	0.20	0.20	0.60	0.20	1.00	0.20	0.20	0.40	0.20
	Best	1.00	0.20	0.20	1.00	1.00	1.00	0.20	0.60	1.00	0.20
	Mean	0.99	0.20	0.20	0.90	0.56	<b>1.00</b>	0.20	0.30	0.86	0.20
	Std	0.04	0.00	0.00	0.14	0.22	0.00	0.00	0.12	0.17	0.00
	$t$ -test	-	1	1	1	1	0	1	1	1	1
F3	Worst	1.00	0.80	0.80	0.80	0.60	0.80	1.00	0.80	0.40	0.00
	Best	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
	Mean	<b>1.00</b>	0.98	0.94	0.98	0.98	0.98	<b>1.00</b>	0.92	0.79	0.00
	Std	0.00	0.07	0.09	0.06	0.09	0.06	0.00	0.10	0.18	0.00
	$t$ -test	-	0	1	0	0	0	0	1	1	1
F4	Worst	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00
	Best	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	0.96
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20
	$t$ -test	-	0	0	0	0	0	0	0	0	0
F5	Worst	1.00	0.25	0.25	0.00	0.25	0.00	0.50	0.75	0.00	0.75
	Best	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	0.25	1.00
	Mean	<b>1.00</b>	0.73	0.69	0.75	0.78	0.21	0.92	0.93	0.08	0.85
	Std	0.00	0.22	0.22	0.23	0.17	0.16	0.14	0.12	0.12	0.13
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F6	Worst	1.00	0.00	0.00	0.50	0.00	0.00	0.50	1.00	0.00	1.00
	Best	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	0.50	1.00
	Mean	<b>1.00</b>	0.72	0.78	0.78	0.74	0.04	0.98	<b>1.00</b>	0.02	<b>1.00</b>
	Std	0.00	0.36	0.29	0.26	0.30	0.14	0.10	0.00	0.10	0.00
	$t$ -test	-	1	1	1	1	1	0	0	1	0
F7	Worst	1.00	0.92	0.88	0.96	0.92	0.96	0.12	0.04	0.28	0.00
	Best	1.00	1.00	1.00	1.00	1.00	1.00	0.28	0.08	0.80	0.16
	Mean	<b>1.00</b>	0.98	0.97	0.99	0.98	<b>1.00</b>	0.21	0.05	0.50	0.04
	Std	0.00	0.03	0.03	0.01	0.03	0.01	0.05	0.02	0.10	0.03
	$t$ -test	-	1	1	0	1	0	1	1	1	1
F8	Worst	0.94	0.67	0.72	0.67	0.72	0.28	0.83	0.50	0.89	0.00

	Best	1.00	1.00	1.00	1.00	1.00	0.72	1.00	0.94	1.00	0.28
	Mean	<b>0.99</b>	0.84	0.86	0.84	0.90	0.47	0.97	0.69	0.98	0.12
	Std	0.02	0.09	0.07	0.07	0.07	0.13	0.04	0.11	0.03	0.07
	<i>t</i> -test	-	1	1	1	1	1	0	1	0	1
	Worst	0.50	0.30	0.30	0.30	0.30	0.10	0.10	0.10	0.40	0.10
F9	Best	0.70	0.50	0.50	0.60	0.50	0.50	0.20	0.50	0.80	0.30
	Mean	<b>0.60</b>	0.37	0.37	0.40	0.38	0.28	0.11	0.28	0.59	0.22
	Std	0.06	0.07	0.06	0.08	0.07	0.11	0.03	0.08	0.10	0.09
	<i>t</i> -test	-	1	1	1	1	1	1	1	0	1
	Worst	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
F10	Best	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00
	Mean	<b>1.00</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	<b>1.00</b>	<b>1.00</b>
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	<i>t</i> -test	-	1	1	1	1	1	1	1	0	0
	Worst	1.00	0.67	0.67	0.67	0.67	0.67	1.00	1.00	0.00	1.00
F11	Best	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Mean	<b>1.00</b>	0.93	0.93	0.92	0.93	0.88	<b>1.00</b>	<b>1.00</b>	0.36	<b>1.00</b>
	Std	0.00	0.14	0.14	0.15	0.14	0.16	0.00	0.00	0.30	0.00
	<i>t</i> -test	-	1	1	1	1	1	0	0	1	0
	Worst	1.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00
F12	Best	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00
	Mean	<b>1.00</b>	0.72	0.88	0.56	0.72	0.08	<b>1.00</b>	0.96	0.00	<b>1.00</b>
	Std	0.00	0.46	0.33	0.51	0.46	0.28	0.00	0.20	0.00	0.00
	<i>t</i> -test	-	1	0	1	1	1	0	0	1	0
	Worst	1.00	0.50	1.00	0.50	0.50	0.00	0.50	1.00	1.00	0.50
F13	Best	1.00	1.00	1.00	1.00	1.00	0.50	1.00	1.00	1.00	1.00
	Mean	<b>1.00</b>	0.98	<b>1.00</b>	0.96	0.98	0.16	0.52	<b>1.00</b>	<b>1.00</b>	0.72
	Std	0.00	0.10	0.00	0.14	0.10	0.24	0.10	0.00	0.00	0.26
	<i>t</i> -test	-	0	1	0	0	1	1	0	0	1
	Worst	1.00	0.00	0.50	0.50	0.50	0.00	0.50	0.50	0.00	0.50
F14	Best	1.00	1.00	0.50	1.00	1.00	0.00	0.50	0.50	0.50	0.50
	Mean	<b>1.00</b>	0.52	0.50	0.88	0.80	0.00	0.50	0.50	0.08	0.50
	Std	0.00	0.23	0.00	0.22	0.25	0.00	0.00	0.00	0.19	0.00
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	1.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.00	0.00
F15	Best	1.00	0.40	0.20	0.40	0.80	0.00	0.20	0.60	0.20	0.60
	Mean	<b>1.00</b>	0.10	0.07	0.18	0.16	0.00	0.20	0.29	0.01	0.48
	Std	0.00	0.13	0.10	0.13	0.21	0.00	0.00	0.16	0.04	0.18
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	1.00	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.00	0.00
<i>t</i> -test summary	Better	-	11	12	10	11	10	8	10	10	10
	Similar	-	4	3	5	4	5	7	5	5	5
	Worst	-	0	0	0	0	0	0	0	0	0

Table IX. Peak ratio (test functions F16 – F30)

Test Func.		LIPS	$r2pso$	$r3pso$	$r2pso-lhc$	$r3pso-lhc$	SPSO	FERPSO	SDE	CDE	SACMA-ES
F16	Worst	0.38	0.00	0.00	0.00	0.00	0.00	0.13	0.13	0.00	0.25
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.25	0.25	0.00	0.25
	Mean	<b>0.46</b>	0.00	0.00	0.00	0.00	0.00	0.14	0.22	0.00	0.25
	Std	0.06	0.00	0.00	0.00	0.00	0.00	0.04	0.05	0.00	0.00
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F17	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.33	0.17	0.17	0.17
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.33	0.33	0.33	0.33
	Mean	<b>0.35</b>	0.00	0.00	0.00	0.00	0.00	0.33	0.20	0.20	0.32
	Std	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.07	0.05
	$t$ -test	-	1	1	1	1	1	0	1	1	0
F18	Worst	0.67	0.00	0.00	0.00	0.00	0.00	0.33	0.17	0.00	0.33
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.50	0.33	0.17	0.50
	Mean	<b>0.67</b>	0.00	0.00	0.00	0.00	0.00	0.42	0.25	0.12	0.45
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.09	0.08	0.08
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F19	Worst	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17
	Mean	<b>0.32</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03
	Std	0.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F20	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.33	0.17	0.17	0.17
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.33	0.33	0.33	0.33
	Mean	<b>0.37</b>	0.00	0.00	0.00	0.00	0.00	0.33	0.22	0.18	0.32
	Std	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.05	0.05
	$t$ -test	-	1	1	1	1	1	0	1	1	0
F21	Worst	0.50	0.00	0.00	0.00	0.00	0.00	0.17	0.17	0.00	0.33
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.33	0.33	0.00	0.50
	Mean	<b>0.62</b>	0.00	0.00	0.00	0.00	0.00	0.20	0.23	0.00	0.43
	Std	0.08	0.00	0.00	0.00	0.00	0.00	0.07	0.09	0.00	0.09
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F22	Worst	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.17	0.33	0.00	0.17
	Mean	<b>0.67</b>	0.00	0.00	0.00	0.00	0.00	0.08	0.17	0.00	0.17
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.14	0.00	0.00
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F23	Worst	0.50	0.00	0.00	0.00	0.00	0.00	0.17	0.17	0.00	0.33
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.33	0.33	0.00	0.50
	Mean	<b>0.53</b>	0.00	0.00	0.00	0.00	0.00	0.25	0.23	0.00	0.38
	Std	0.07	0.00	0.00	0.00	0.00	0.00	0.09	0.09	0.00	0.08
	$t$ -test	-	1	1	1	1	1	1	1	1	1

	Worst	0.50	0.00	0.00	0.00	0.00	0.00	0.17	0.17	0.00	0.17
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.33	0.50	0.00	0.33
F24	Mean	<b>0.58</b>	0.00	0.00	0.00	0.00	0.00	0.25	0.30	0.00	0.28
	Std	0.09	0.00	0.00	0.00	0.00	0.00	0.09	0.11	0.00	0.08
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.17	0.17	0.00	0.17
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.33	0.33	0.00	0.33
F25	Mean	<b>0.35</b>	0.00	0.00	0.00	0.00	0.00	0.18	0.18	0.00	0.23
	Std	0.05	0.00	0.00	0.00	0.00	0.00	0.05	0.05	0.00	0.09
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.00
	Best	0.83	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.17
F26	Mean	<b>0.68</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	0.12
	Std	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.00	0.08
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	0.50	0.00	0.00	0.00	0.00	0.00	0.17	0.17	0.00	0.17
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.33	0.50	0.00	0.33
F27	Mean	<b>0.58</b>	0.00	0.00	0.00	0.00	0.00	0.27	0.27	0.00	0.28
	Std	0.09	0.00	0.00	0.00	0.00	0.00	0.09	0.14	0.00	0.08
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Best	0.40	0.00	0.00	0.00	0.00	0.00	0.10	0.20	0.00	0.30
F28	Mean	<b>0.40</b>	0.00	0.00	0.00	0.00	0.00	0.03	0.09	0.00	0.14
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.06	0.00	0.12
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	0.10	0.00	0.00	0.00	0.00	0.00	0.10	0.10	0.00	0.10
	Best	0.10	0.00	0.00	0.00	0.00	0.00	0.10	0.10	0.00	0.10
F29	Mean	<b>0.10</b>	0.00	0.00	0.00	0.00	0.00	<b>0.10</b>	<b>0.10</b>	0.00	<b>0.10</b>
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	<i>t</i> -test	-	1	1	1	1	1	0	0	1	0
	Worst	0.40	0.00	0.00	0.00	0.00	0.00	0.10	0.10	0.00	0.10
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.20	0.20	0.00	0.30
F30	Mean	<b>0.48</b>	0.00	0.00	0.00	0.00	0.00	0.12	0.16	0.00	0.20
	Std	0.04	0.00	0.00	0.00	0.00	0.00	0.04	0.05	0.00	0.07
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
<i>t</i> -test summary	Better	-	15	15	15	15	15	12	14	15	12
	Similar	-	0	0	0	0	0	3	1	0	3
	Worst	-	0	0	0	0	0	0	0	0	0

Table X Peak ratio on high dimensional problems (F16 - F20)

Test Func.		LIPS	$r2pso$	$r3pso$	$r2pso-lhc$	$r3pso-lhc$	SPSO	FERPSO	SDE	CDE	SACMA-ES
F16 (20D)	Worst	0.38	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.25
	Best	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.25
	Mean	<b>0.46</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.13	0.00	0.25
	Std	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F17 (20D)	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.33
	Best	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.33
	Mean	<b>0.33</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.22	0.00	<b>0.33</b>
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.00
	$t$ -test	-	1	1	1	1	1	1	1	1	0
F18 (20D)	Worst	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.33
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.50
	Mean	<b>0.65</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.45
	Std	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.00	0.08
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F19 (20D)	Worst	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Best	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Mean	<b>0.32</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Std	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$t$ -test	-	1	1	1	1	1	1	1	1	1
F20 (20D)	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.17	0.00	0.17
	Best	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00	0.33
	Mean	<b>0.33</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.32
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.00	0.05
	$t$ -test	-	1	1	1	1	1	1	1	1	0
F16 (30D)	Worst	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.25
	Best	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38
	Mean	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	<b>0.26</b>
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04
	$t$ -test	-	1	1	1	1	1	1	1	1	0
F17 (30D)	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33
	Best	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33
	Mean	<b>0.33</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	<b>0.33</b>
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$t$ -test	-	1	1	1	1	1	1	1	1	0
F18 (30D)	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33
	Best	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50
	Mean	<b>0.50</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.35
	Std	0.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05
	$t$ -test	-	1	1	1	1	1	1	1	1	0

	<i>t</i> -test	-	1	1	1	1	1	1	1	1	1
	Worst	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F19	Best	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
(30D)	Mean	<b>0.00</b>									
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	<i>t</i> -test	-	0	0	0	0	0	0	0	0	0
	Worst	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33
F20	Best	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.33
(30D)	Mean	<b>0.33</b>	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	<b>0.33</b>
	Std	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	<i>t</i> -test	-	1	1	1	1	1	1	1	1	0
	Better	-	9	9	9	9	9	9	9	9	4
<i>t</i> -test summary	Similar	-	1	1	1	1	1	1	1	1	6
	Worst	-	0	0	0	0	0	0	0	0	0