

# **Problem Definitions and Evaluation Criteria for the CEC 2021 on Multimodal Multiobjective Path Planning Optimization**

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In multiobjective optimization problems, there may exist two or more global or local Pareto optimal sets (PSs) and some of them may correspond to the same Pareto Front (PF). These problems are defined as multimodal multiobjective optimization problems (MMOPs) [1-2].

Path planning problems are representative MMOPs. Fig.1 shows a simple example of the path planning optimization problem. Black areas are feasible paths, white areas are infeasible regions and red areas are congested regions. There are two objectives to be optimized. One is the length of the path and the other is the number of red areas along the path. Fig. 2 shows all the possible paths in the objective space. As shown, the length of the circled one is the shortest and there is one red area in it. In fact, there are three different paths corresponding to the circled dot. These three paths are shown in Fig. 3. Arguably, finding one of them may be sufficient to obtain an acceptable solution. However, failing to identify more than one of the shortest paths may prevent the decision-maker from considering solution options that could bring about improved performance [3-6]. For example, a traveler wants to go along the yellow area in Fig.4. However, only the first or second path is provided, the best path will be missed. Actually, different decision-makers have different preferences, so providing multiple excellent solutions is necessary. In recent years, there are few researchers constructing related test problems. These test problems are needed urgently to resemble complicated real-life problems and thus promoting the development of multimodal multiobjective path planning optimization.

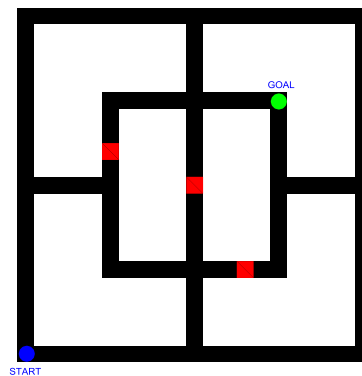


Fig. 1. An example map

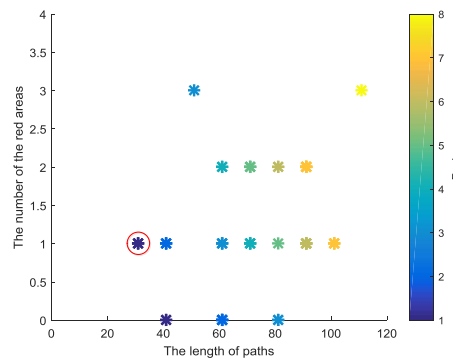


Fig. 2. The distribution of all possible paths in the objective space

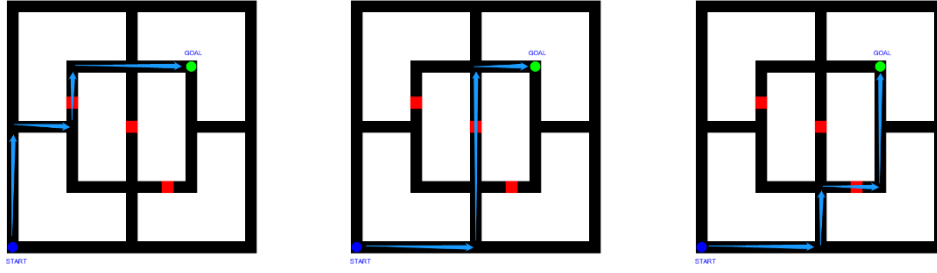


Fig. 3. The three shortest paths with the same number of red areas

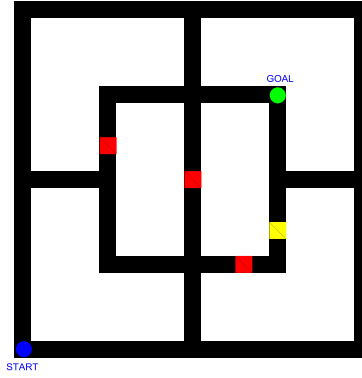


Fig. 4. The map with preference setting (the yellow area)

In the multimodal multiobjective path planning (MMOPP) test suite of CEC'2021, a set of MMOPP test problems with different characters are designed, and they are similar to the real-life problems. It is well known that the main roads of big cities in China have distinct characteristics, such as circle traffic networks, long straight roads, complex alleys. Fig. 5 shows the real main roads of Beijing, Zhengzhou, Chengdu. The maps of this test suite are created by simulating these characteristics Fig. 6 shows an example in this test suite. Twelve problems in three different scenarios are constructed. In addition, a fair and appropriate evaluation criterion is given to assess the performance of different algorithms.

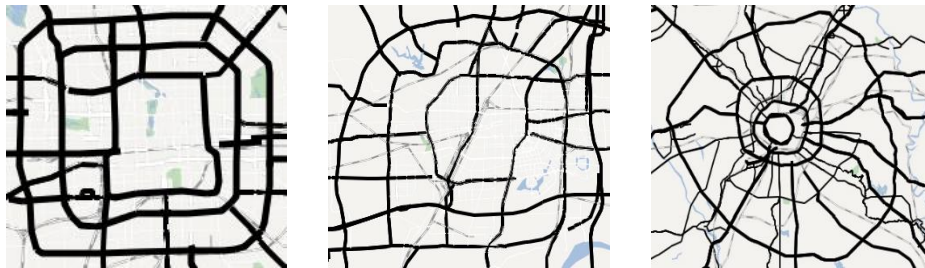


Fig. 5. The main roads of real cites

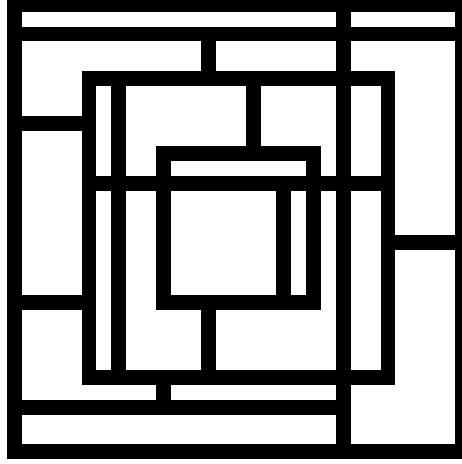


Fig. 6. The simulated map

The Matlab codes for the MMOPP test suite of CEC'2021 can be downloaded from the website given below:

<http://www5.zzu.edu.cn/ecilab/info/1036/1251.htm>

## 1 Introduction to the CEC'2021 MMOPP test problems

### 1.1 Some Definitions

**The introduction for the map:** The map is simulated by a matrix consisted of 0 or 1. The area where 0 is indicated is passable, and the area where 1 is indicated is not passable. The rows of this

matrix are  $x$ , and the columns of this matrix are  $y$ . For example,  $map = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ , and Fig 7 is

its visual image. The black areas are passable, while the white areas are not passable.

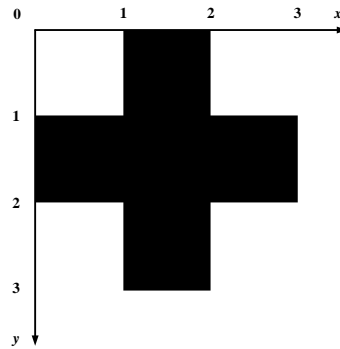


Fig. 7. A simple map example

**The rules:** It is allowed to walk in up, down, left, right directions, and it is not allowed to go the repetitive ways, like Fig. 8. Moreover, it is forbidden to go beyond the boundaries of the map.

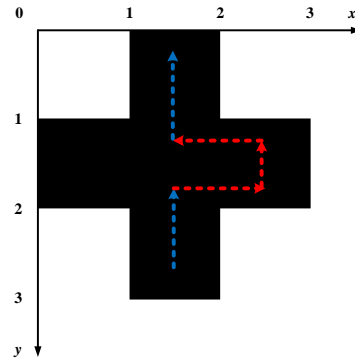


Fig. 8. The repetitive way

**The passable path:** A passable path should start from 'START', finish at 'GOAL', and every step of the path is passable. Coordinate values of every step in walking order are used to represent this path. The passable path in Fig. 9 can be expressed by  $[2,3;2,2;2,1]$ . The blue area in Figure 10 is the area represented by  $[1,2]$ .

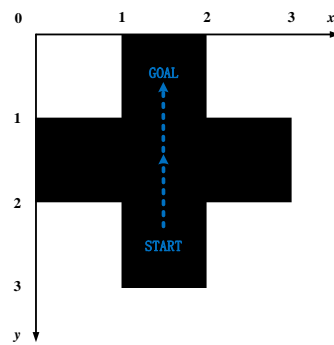


Fig. 9. A passable path

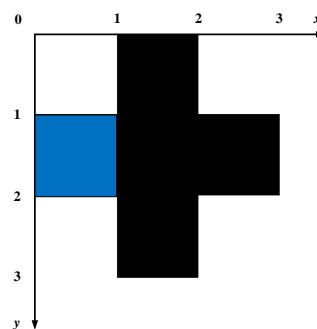


Fig. 10. The area of  $[1,2]$

**All the objectives need to be minimized.**

## 1.2 Definitions of the CEC'2021 MMOPP test problems

### 1.2.1 The first kind of test problems

The first kind of MMOPP test problems is introduced in this subsection. In real life, traffic jams are common occurrences. In the following test problems, some red areas are designed to simulate traffic jams. Besides, the number of intersections of a path is also an important indicator when people choose a path.

Note that both T-junction and intersection are considered as the intersection in these problems.

#### Test problem 1

**The introduction of this map:** It is made up of three circles of different sizes, and some straight paths of different lengths. Many red areas have been placed to simulate the severe congestion of a city.

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The red areas' number of the passable path.

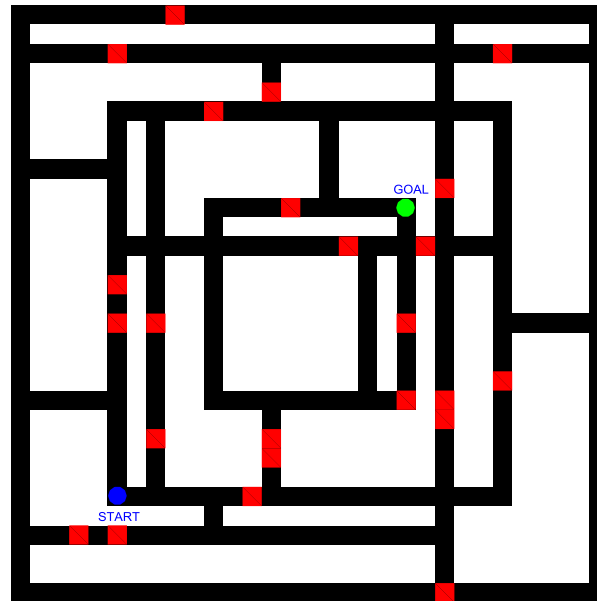


Fig. 11. Test problem 1

#### Test problem 2

**The introduction of this map:** This map is similar to test problem 1.1 in complexity, whereas the differences are that more goals are needed to optimize, and 'START' has been changed.

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;

2.  $f_2$ : The red areas' number of the passable path;
3.  $f_3$ : The intersection number of the passable path.

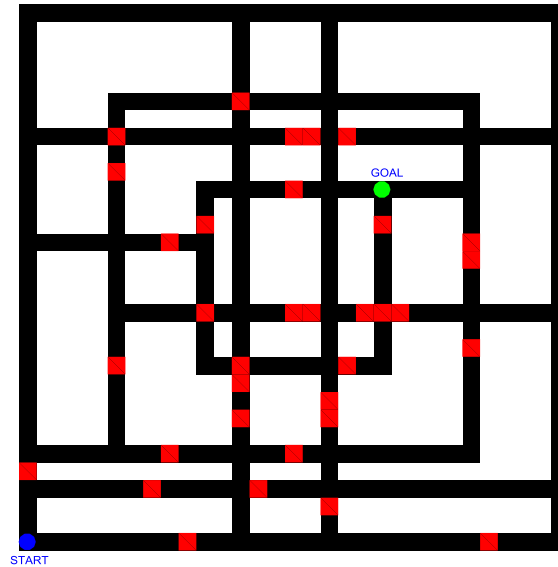


Fig. 12. Test problem 2

### Test problem 3

**The introduction of this map:** This map consists of four circles and many straight paths. In addition, some paths that are not completely connected with other paths are added.

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The red areas' number of the passable path;
3.  $f_3$ : The intersection number of the passable path.

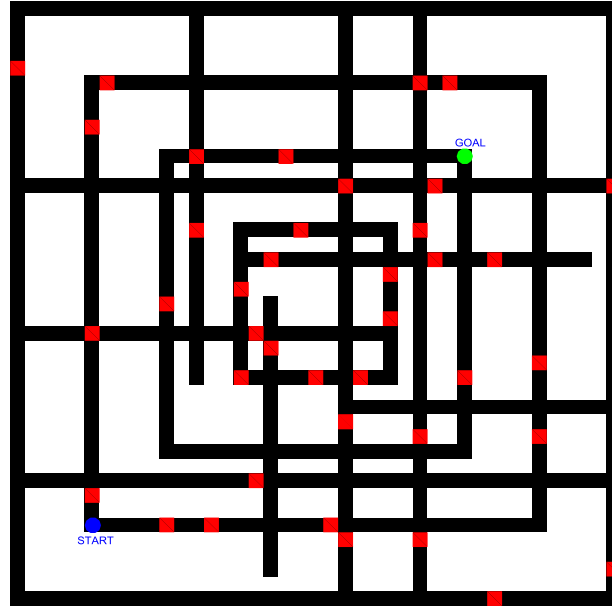


Fig. 13. Test problem 3

#### Test problem 4

**The introduction of this map:** This map consists of four circles and many straight paths. Many paths that are not completely connected with other paths are added.

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The red areas' number of the passable path;
3.  $f_3$ : The intersection number of the passable path.



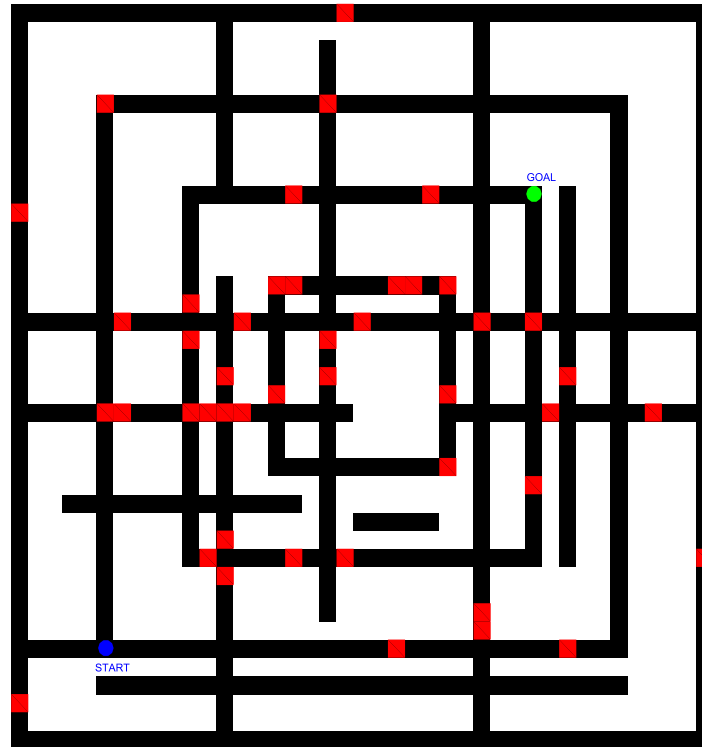


Fig. 14. Test problem 4

### Test problem 5

**The introduction of this map:** This map is more complex, consisted of six circles, and many straight paths.

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The red areas' number of the passable path;
3.  $f_3$ : The intersection number of the passable path.

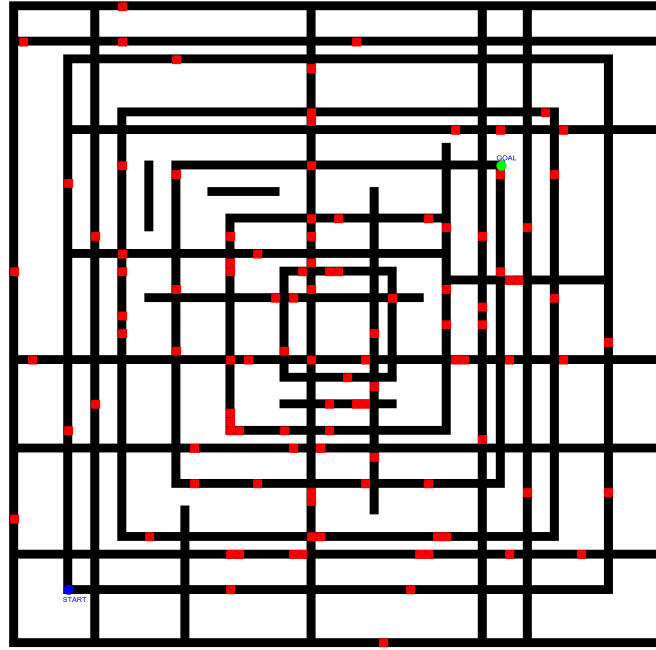


Fig. 15. Test problem 5

### 1.2.2 The second kind of test problems

The second kind of MMOPP is introduced in this subsection. In the first kind of test problem, the congestion in the city is simply simulated, whereas it is too simple to represent more features in real path planning. In the following problems, different values are assigned to every passable area, which can simulate more complex path planning problems. In Fig.16, the values on each point can represent the congestion degree or road width.

Note that: Since several values are assigned to the same point in some test problems, only the map is given in this report. The values corresponding to each map is given in the test problem data (F).

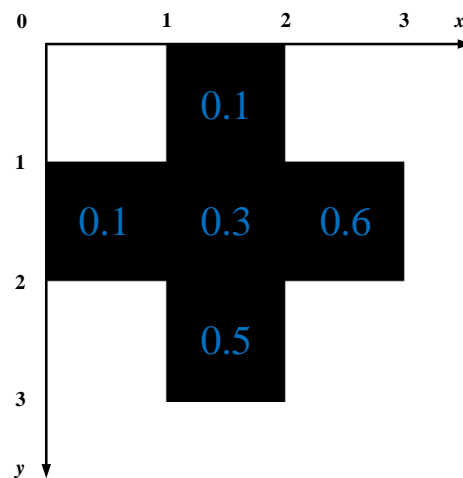


Fig. 16. A simple example

## Test problem 6

The objectives need to be minimized:

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The cumulative  $F_1$  value of the passable path. The values of  $F_1$  are getting larger from the outside to the inside. In Fig. 17, if the passable path is  $[1,2; 2,2; 3,2]$ , the  $f_2$  of this path is  $0.1 + 0.3 + 0.1 = 0.5$ .

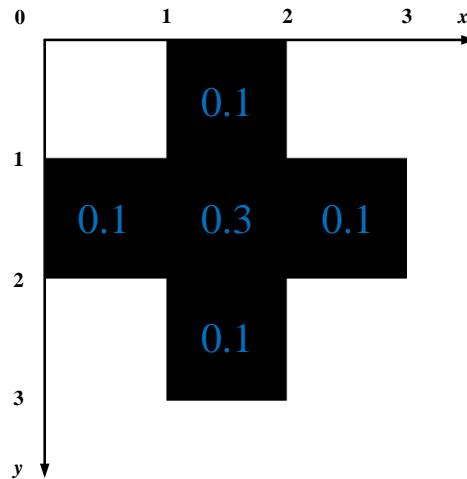


Fig. 17. A simple show of  $F_1$  values

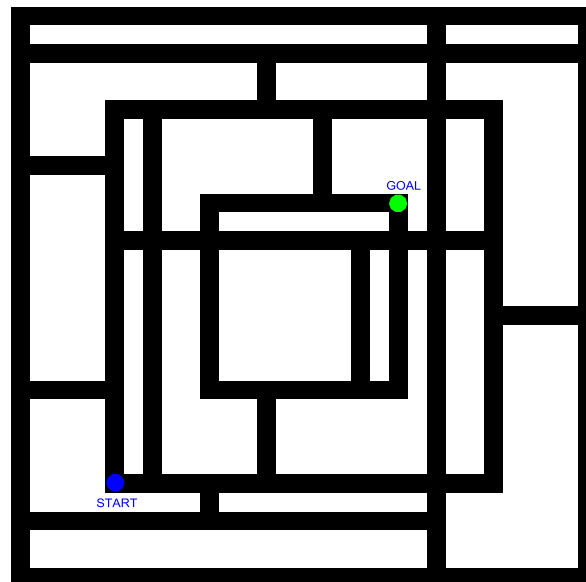


Fig. 18. Test problem 6

## Test problem 7

The objectives need to be minimized:

1.  $f_1$ : The length of the passable path;

2.  $f_2$ : The cumulative  $F_1$  value of the passable path. From the outside to the inside, the values of  $F_1$  are getting larger;
3.  $f_3$ : The cumulative  $F_2$  value of the passable path. From the outside to the inside, the values of  $F_2$  are getting smaller, like Fig. 19.

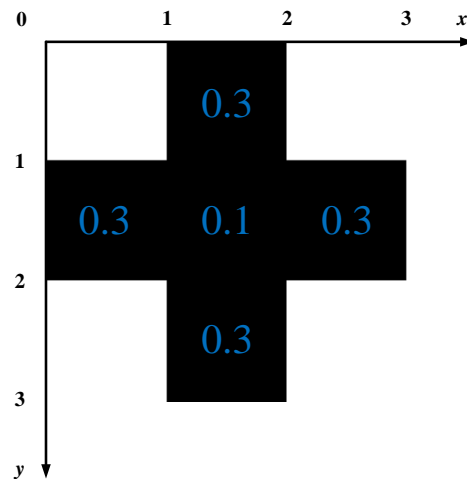


Fig. 19. A simple show of  $F_2$  values

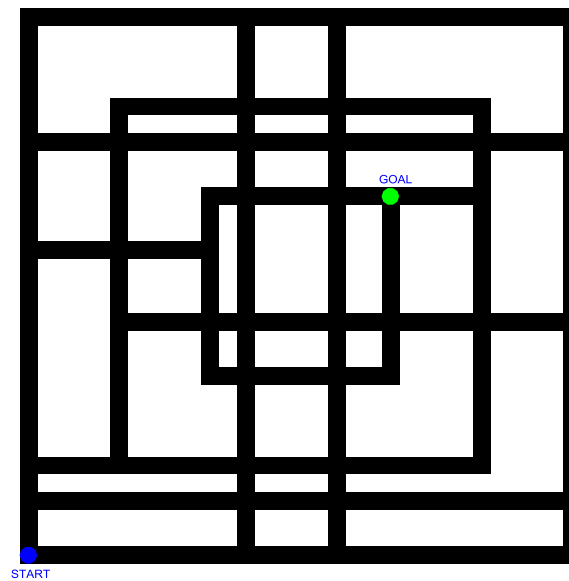


Fig. 20. Test problem 7

## Test problem 8

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The cumulative  $F_1$  value of the passable path. From the outside to the inside, the values of  $F_1$  are getting larger;
3.  $f_3$ : The cumulative  $F_2$  value of the passable path. From the outside to the inside, the

values of  $F_2$  are getting smaller;

4.  $f_4$ : The cumulative  $F_3$  value of the passable path. From the up to the down, the values of  $F_3$  are getting larger, like Fig. 21.

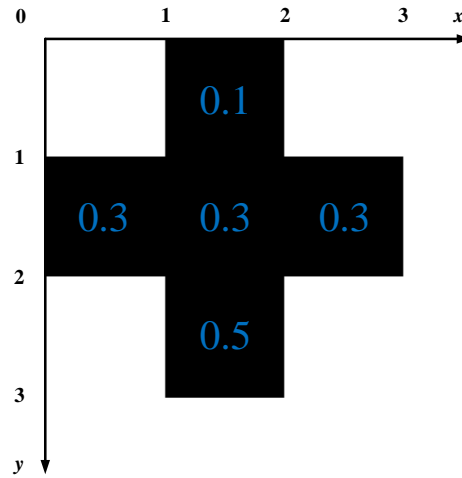


Fig. 21. A simple show of  $F_3$  values

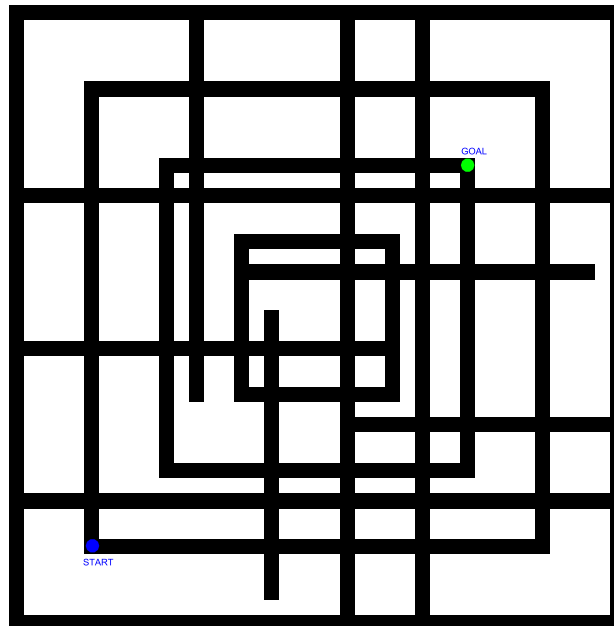


Fig. 22. Test problem 8

## Test problem 9

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The cumulative  $F_1$  value of the passable path. From the outside to the inside, the values of  $F_1$  are getting larger;

3.  $f_3$ : The cumulative  $F_2$  value of the passable path. From the outside to the inside, the values of  $F_2$  are getting smaller;
4.  $f_4$ : The cumulative  $F_3$  value of the passable path. From the up to the down, the values of  $F_3$  are getting larger;
5.  $f_5$ : The cumulative  $F_4$  value of the passable path. From the up to the down, the values of  $F_4$  are getting larger, like Fig. 23.

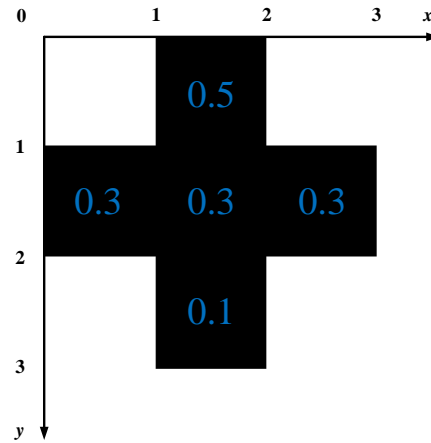


Fig. 23. A simple show of  $F_4$  values

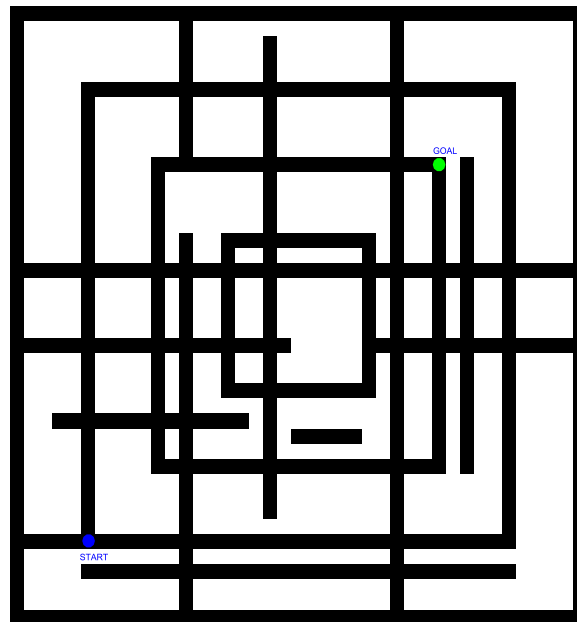


Fig. 24. Test problem 9

## Test problem 10

The objectives need to be minimized:

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The cumulative  $F_1$  value of the passable path. From the outside to the inside, the

- values of  $F_1$  are getting larger;
3.  $f_3$ : The cumulative  $F_2$  value of the passable path. From the outside to the inside, the values of  $F_2$  are getting smaller;
  4.  $f_4$ : The cumulative  $F_3$  value of the passable path. From the up to the down, the values of  $F_3$  are getting larger;
  5.  $f_5$ : The cumulative  $F_4$  value of the passable path. From the up to the down, the values of  $F_4$  are getting larger.
  6.  $f_6$ : The cumulative  $F_5$  value of the passable path. From the left to the right, the values of  $F_5$  are getting larger, like Fig. 25;
  7.  $f_7$ : The cumulative  $F_6$  value of the passable path. From the left to the right, the values of  $F_6$  are getting smaller, like Fig. 26.

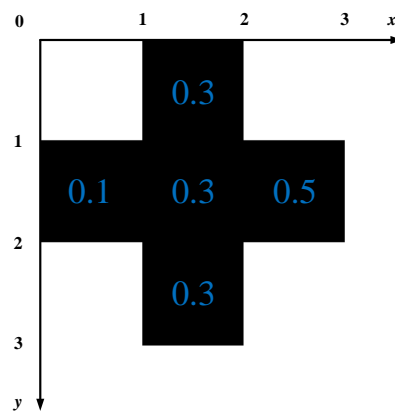


Fig. 25. A simple show of  $F_5$  values

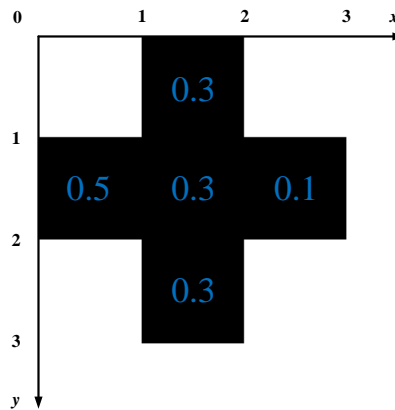


Fig. 26. A simple show of  $F_6$  values

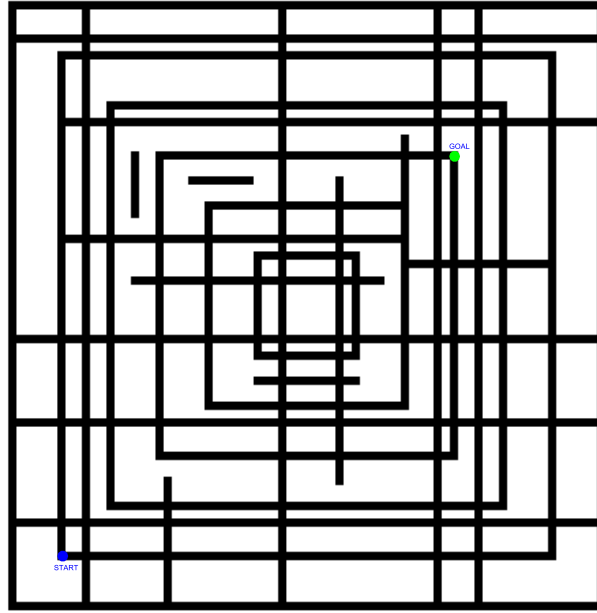


Fig. 27. Test problem 10

### 1.2.3 The third kind of test problems

The third kind of MMOPP is introduced in this subsection. In some special situations, the path has to go through some places except for the 'START' and the 'GOAL'. The following maps are designed to simulate these special situations. In these maps, yellow areas are necessary to go through. For example, although the path of Fig. 28 goes through the 'START' and the 'GOAL', it is not a feasible path because it doesn't go through the yellow area. One right path is shown in Fig. 29.

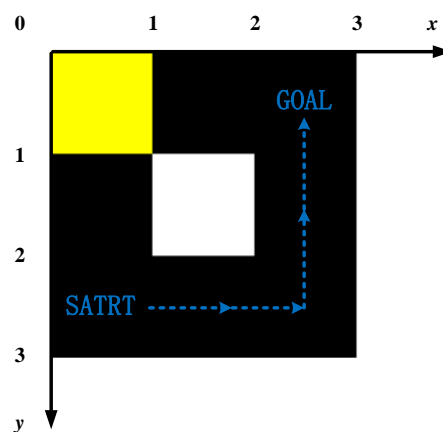


Fig. 28. A wrong path



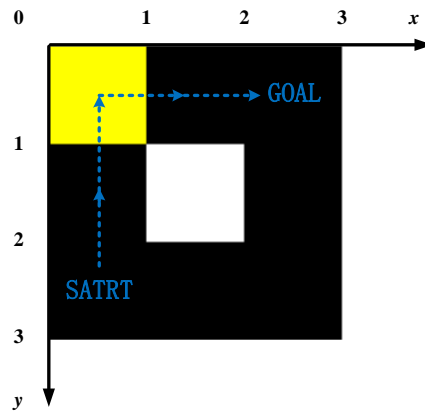


Fig. 29. A right path

### Test problem 11

**The number of yellow areas: 1**

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;
2.  $f_2$ : The cumulative  $F_1$  value of the passable path. From the outside to the inside, the values of  $F_1$  are getting larger.

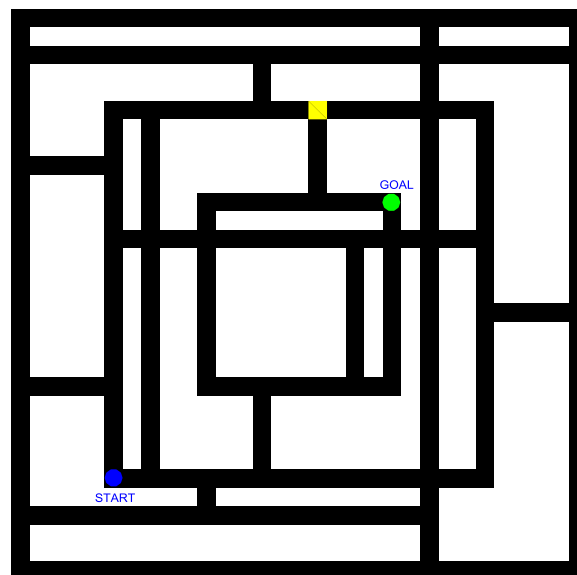


Fig. 30. Test problem 11

### Test problem 12

**The number of yellow areas: 2**

**The objectives need to be minimized:**

1.  $f_1$ : The length of the passable path;

2.  $f_2$ : The cumulative  $F_1$  value of the passable path. From the outside to the inside, the values of  $F_1$  are getting larger;
3.  $f_3$ : The cumulative  $F_2$  value of the passable path. From the outside to the inside, the values of  $F_2$  are getting smaller.

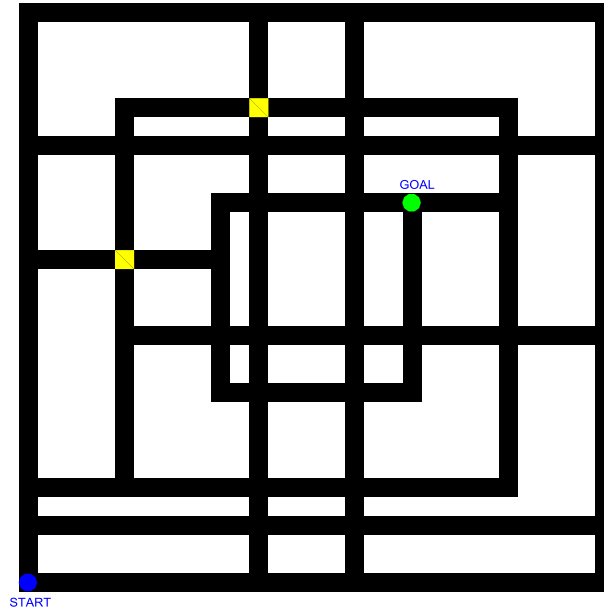


Fig. 31. Test problem 12

## 2 Evaluation criteria

### 2.1 Performance indicators

Participants need to submit PFs and PSs obtained by their algorithms. The results from different participants are compared as follows.

If a PF dominates the other one completely, the former one will win, as shown in Fig. 32.

If a PF can't dominate the other one completely, as shown in Fig. 33, they will be compared in the following way. These PFs are combined and a new PF is obtained. The point in the new PF is assigned 1. The number of paths corresponding to the point in the new PF is regarded as an additional score. One path contributes to 0.3. In Fig. 33, the loser's score =  $4 \times 1.0 + (1 + 2 + 3 + 4) \times 0.3 = 7.0$ , and the winner's score =  $3 \times 1.0 + (5 + 6 + 7) \times 0.3 = 8.4$ .

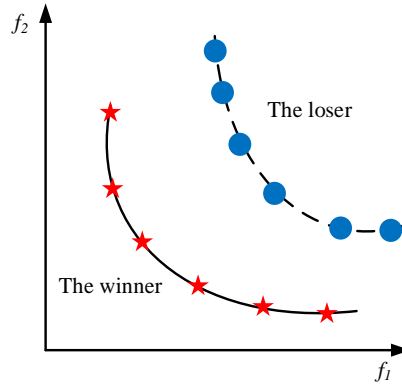


Fig. 32. Dominating completely

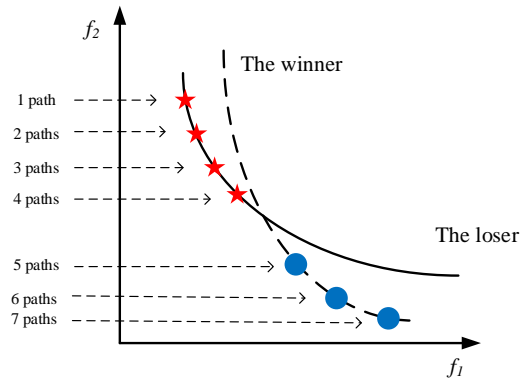


Fig. 33. Not dominating completely

## 2.2 Experimental setting

**Problems:** 12 MMOPP problems

**The objectives need to be minimized:** Different test problems have different objectives to be optimized.

**The constraint:** Every path is passable, and the path must go through the yellow areas for test problems 11, 12.

**The exhaust algorithm is not allowed.**

**The data of test problems are available at:**

<http://www5.zzu.edu.cn/ecilab/info/1036/1251.htm>

## 2.3 Algorithm Complexity

a) Run the test program below:

```

x = 0.55
for i = 1: 1000000
    x = x + x; x = x / 2; x = x * x; x = sqrt(x); x = log(x); x = exp(x); x = x / (x + 2);

```

end

Computing time for the above =  $T_0$ ;

b) Evaluate the computing time for test problem 1, 2, ...12, it gives  $T_1, T_2, \dots, T_{12}$ ;

c) Calculate  $T_1/T_0, T_2/T_0, \dots, T_{12}/T_0$ .

The complexity of the algorithm is reflected by  $T_1/T_0, T_2/T_0, \dots, T_{12}/T_0$ .

### 3 Results Format

The results, codes and the description for your algorithm need to be submitted. The organizers will present an overall analysis and comparison based on these results.

Please pack all the required files in one zip document named “AlgorithmName\_TeamName.zip”. Please record the results into tables named Table I.xls, Table II.xls, ...Table XIV.xls. Two examples are shown in Table XV and Table XVI. ‘Path1\_m\_n’, ‘1’ represents test problem 1, ‘m\_n’ represents the  $m^{\text{th}}$  path to the  $n^{\text{th}}$  path. Note that 0 is used to supplemented short paths when short paths and long paths are put together.

Table I. Results of MMOPP

The test problem	The Pareto optimal paths	The objective values	Number of paths
Test problem 1	Path1_1_n	$f_1, f_2$	n
	Path1_n+1_n+m	$f_1, f_2$	m
	Path1_n+m+1_n+m+q	$f_1, f_2$	q
	...	$f_1, f_2$	...
Test problem 2	Path2_1_n	$f_1, f_2, f_3$	n
	Path2_n+1_n+m	$f_1, f_2, f_3$	m
	Path2_n+m+1_n+m+q	$f_1, f_2, f_3$	q
	...	$f_1, f_2, f_3$	...
...	...	$f_1, f_2, f_3$	...
Test problem 5	Path5_1_n	$f_1, f_2, f_3$	n
	...	$f_1, f_2, f_3$	...
Test problem 6	Path6_1_n	$f_1, f_2$	n
	...	$f_1, f_2$	...
Test problem 7	...	$f_1, f_2, f_3$	...
Test problem 8	...	$f_1, f_2, f_3, f_4$	...
Test problem 9	...	$f_1, f_2, f_3, f_4, f_5$	...
Test problem 10	...	$f_1, f_2, f_3, f_4, f_5, f_6, f_7$	...
Test problem 11	...	$f_1, f_2$	...
Test problem 12	...	$f_1, f_2, f_3$	...

Table II. The Pareto optimal paths of test problem 1

Path1_1 (x)	Path1_1 (y)	Path1_2(x)	Path1_2(y)	...	Path1_end(x)	Path1_end(y)
10	30	10	30	...	10	30
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...

Table III. The Pareto optimal paths of test problem 2

Path2_1 (x)	Path2_1 (y)	Path2_2(x)	Path2_2(y)	...	Path2_end(x)	Path2_end(y)
5	35	5	35	...	5	35
...	...	...	...	...	...	...
...	...	...	...	...	...	...

Table IV. The Pareto optimal paths of test problem 3

...

Table V. The Pareto optimal paths of test problem 4

...

Table VI. The Pareto optimal paths of test problem 5

...

Table VII. The Pareto optimal paths of test problem 6

...

Table VIII. The Pareto optimal paths of test problem 7

...

Table IX. The Pareto optimal paths of test problem 8

...

Table X. The Pareto optimal paths of test problem 9

...

Table XI. The Pareto optimal paths of test problem 10

...

Table XII. The Pareto optimal paths of test problem 11

...

Table XIII. The Pareto optimal paths of test problem 12

...

Table XIV. Computational Complexity

$T_0$	$T_1/T_0$	$T_2/T_0$	...	$T_{12}/T_0$

Table XV. An example of MMOPP results

The test problem	The Pareto optimal paths	The objective values	Number of paths
Test problem 1	Path1_1_5	10, 3	5
	Path1_6_12	12, 2	7
Test problem 2	Path2_1_6	15, 3, 5	6
	Path2_7_15	15, 5, 3	9

...	...	...	...
Test problem 12	Path12_1_3	15, 12.8, 13.5	3
	Path12_4_6	14, 13.5, 12.8	3

Table XVI. An example of test problem 1's Pareto optimal paths

Path1_1 (x)	Path1_1 (y)	Path1_2(x)	Path1_2(y)	...	Path1_end(x)	Path1_end(y)
10	30	10	30	...	10	30
11	30	10	29	...	11	30
12	30	10	28	...	12	30
13	30	10	27	...	13	30
14	30	10	26	...	14	30
15	30	10	25	...	15	30
16	30	10	24	...	16	30
17	30	10	23	...	17	30
18	30	10	22	...	18	30
18	29	10	21	...	18	29
18	28	10	20	...	18	28
18	27	10	19	...	18	27
18	26	10	18	...	18	26
18	25	10	17	...	18	25
19	25	11	17	...	17	25
20	25	12	17	...	16	25
21	25	13	17	...	15	25
22	25	14	17	...	15	24
23	25	15	17	...	15	23
24	25	15	16	...	15	22
25	25	15	15	...	15	21
25	24	16	15	...	15	20
25	23	17	15	...	15	19
25	22	18	15	...	15	18
25	21	19	15	...	15	17
25	20	20	15	...	15	16
25	19	21	15	...	15	15
25	18	22	15	...	16	15
25	17	23	15	...	17	15
25	16	24	15	...	18	15
25	15	25	15	...	19	15
10	30	10	30	...	20	15
11	30	10	29	...	21	15
12	30	10	28	...	22	15
13	30	10	27	...	23	15
14	30	10	26	...	24	15
15	30	10	25	...	25	15

16	30	10	24	...	10	30
17	30	10	23	...	11	30
18	30	10	22	...	12	30
18	29	10	21	...	13	30
18	28	10	20	...	14	30
18	27	10	19	...	15	30
18	26	10	18	...	16	30
18	25	10	17	...	17	30
19	25	11	17	...	18	30
20	25	12	17	...	18	29
21	25	13	17	...	18	28
22	25	14	17	...	18	27
23	25	15	17	...	18	26
24	25	15	16	...	18	25
25	25	15	15	...	17	25
25	24	16	15	...	16	25
25	23	17	15	...	15	25
25	22	18	15	...	15	24
25	21	19	15	...	15	23
25	20	20	15	...	15	22
25	19	21	15	...	15	21
25	18	22	15	...	15	20
25	17	23	15	...	15	19
25	16	24	15	...	15	18
25	15	25	15	...	15	17
10	30	10	30	...	15	16
11	30	10	29	...	15	15
12	30	10	28	...	16	15
13	30	10	27	...	17	15
14	30	10	26	...	18	15
15	30	10	25	...	19	15
16	30	10	24	...	20	15
17	30	10	23	...	21	15
18	30	10	22	...	22	15
18	29	10	21	...	23	15
18	28	10	20	...	24	15
18	27	10	19	...	25	15
18	26	10	18	...	10	30
18	25	10	17	...	11	30
19	25	11	17	...	12	30
20	25	12	17	...	13	30
21	25	13	17	...	14	30
22	25	14	17	...	15	30

23	25	15	17	...	16	30
24	25	15	16	...	17	30
25	25	15	15	...	18	30
25	24	16	15	...	18	29
25	23	17	15	...	18	28
25	22	18	15	...	18	27
25	21	19	15	...	18	26
25	20	20	15	...	18	25
25	19	21	15	...	17	25
25	18	22	15	...	16	25
25	17	23	15	...	15	25
25	16	24	15	...	15	24
25	15	25	15	...	15	23
0	0	0	0	...	15	22
0	0	0	0	...	15	21
0	0	0	0	...	15	20
0	0	0	0	...	15	19
0	0	0	0	...	15	18
0	0	0	0	...	15	17

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